Error estimates of a theta-scheme for second-order mean field games

J. Frédéric BONNANS, Inria at L2S, Université Paris-Saclay - Gif-sur-Yvette

Kang LIU, CMAP, École polytechnique, CNRS, IP Paris - Palaiseau

Laurent PFEIFFER, Inria at L2S, Université Paris-Saclay - Gif-sur-Yvette

We introduce and analyze a new finite-difference scheme, that we call θ-scheme, for solving monotone second-order mean field games. These games consist of a coupled system of the Fokker-Planck and the Hamilton-Jacobi-Bellman equation. We use the θ-method for discretizing the diffusion terms, that is to say, we approximate them with a convex combination of an implicit and an explicit term. In contrast, we use an explicit centered scheme for the first-order terms. Assuming that the running cost is strongly convex and regular, we first prove the monotonicity and the stability of our θ-scheme, under a CFL condition. Taking advantage of the regularity of the solution of the continuous problem, we estimate the consistency error of the θ-scheme. Our main result is a convergence rate of order $\mathcal{O}(h^r)$ for the θ-scheme, where h is the step length of the space variable and $r \in (0, 1)$ is related to the Hölder continuity of the solution (and some of its derivatives) of the continuous problem.

Contact : kang.liu@polytechnique.edu