Entire solutions to Allen-Cahn with triple junction structure at infinity

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Consider a smooth potential $W : R^2 \to [0,\infty)$ vanishing at exactly three points (wells), say p_1, p_2 and p_3. We seek an entire solution $u : R^2 \to R^2$ to the PDE

$$\Delta u = \nabla_u W(u) \quad (*)$$

having triple junction structure at infinity. This has been achieved by Bronsard, Gui and Schatzman under an assumption of equivariance of the potential and much more recently by Fusco under a less stringent symmetry assumption. Here we make no symmetry assumption whatsoever. Under a non-degeneracy assumption on the Hessian matrix of W at the three wells, along with an assumption of a triangle inequality holding between the three wells with respect to the (degenerate) metric having conformal factor $\sqrt{W}$, we prove the following :

Théorème 1. *There exists an entire solution u to $(*)$ that is locally minimizing (i.e. on compact sets) with respect to the energy*

$$E(u) := \int W(u) + \frac{1}{2}|\nabla u|^2 \, dx$$

Furthermore, along subsequences, the 'blow-downs' $\{u_R\}$ converge as $R \to \infty$ to a minimal partition of space. Here $u_R(x) := u(Rx)$ and by 'minimal partition' we mean a function taking only the values p_1, p_2 and p_3, with $\{u(x) = p_\ell\}$ for $\ell = 1, 2, 3$ consisting of a single wedge centered at the origin with opening angle determined by W.

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