A symmetry breaking phenomenon for anisotropic harmonic maps into the circle

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Consider, in the annulus $A_\rho = \{\rho < |x| < 1\} \subset \mathbb{R}^2$ $(0 < \rho < 1)$, the energy of maps $u\colon A_\rho \to \mathbb{S}^1$ given by

$$E_\delta(u; A_\rho) = \int_{A_\rho} \left[(1+\delta)(\operatorname{div} u)^2 + (1-\delta)(\operatorname{curl} u)^2 \right] dx,$$

where $0 < |\delta| < 1$ is an anisotropy parameter which appears in various liquid crystal models. We investigate the minimality, with respect to their own boundary values on ∂A_ρ, of homogeneous critical points $u(re^{i\theta}) = \xi(\theta)$ with a given winding number $d \in \mathbb{Z}$. We prove uniqueness properties of such critical points, and show, for $d \notin \{0, 1, 2\}$, that they are minimizing in thin annuli $\rho > \rho_*$ but non-minimizing in thick annuli $\rho < \rho_*$. This has consequences on the asymptotic behavior of entire solutions to an anisotropic Ginzburg-Landau equation : they behave quite differently from the isotropic case $\delta = 0$, for instance with respect to the quantization properties discovered by Brezis, Merle and Rivière.

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