DE LA RECHERCHE À L'INDUSTRIE

Vibrations of two coaxial flexible cylinders in a viscous fluid

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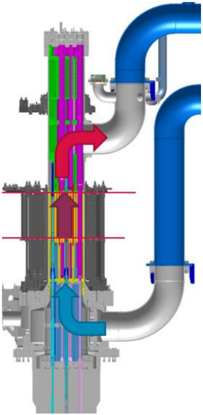
FSI problem of a body vibrating in a fluid

- Mechanics of plants and trees
- Understanding of animal swimming
- Energy harvesting from a flexible structure
- **Nuclear technology**
- etc.



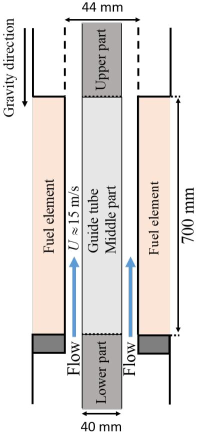


FIGURE – Left : sketch of the **Jules Horowitz Reactor (JHR)**. Right : axial cross section of the JHR assembly cell.

Definition of the problem

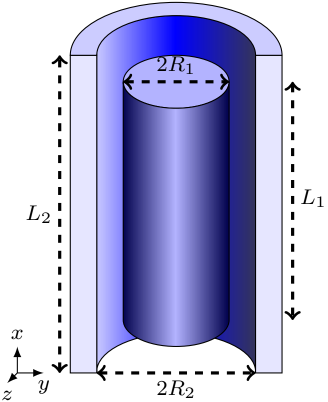


FIGURE – Schematic diagram of the system

- Two coaxial oscillating cylinders $\mathcal{C}_j$ with radii R_j and length L
- Immersed in a fluid of kinematic viscosity ν
- Imposed displacement $\Re\left\{\mathrm{e}^{\mathrm{i}\Omega_i T}\mathbf{Q}_i\left(X\right)\right\}$

$$\mathbf{Q}_i(X) = Q_i\, W_i\left(X\right)\mathbf{e}_y,$$

Ω_i the angular frequency,
Q_i the amplitude of the displacement,
$W_i(X)$ the bending mode of vibration of an Euler-Bernoulli beam

$$W_i\left(X\right) = \chi_i^{(1)}\cosh\left(\Lambda_i X\right) + \chi_i^{(2)}\cos\left(\Lambda_i X\right) + \chi_i^{(3)}\sinh\left(\Lambda_i X\right) + \chi_i^{(4)}\sin\left(\Lambda_i X\right).$$

- **Small amplitude** of the displacement, i.e. $Q_i \ll R_2 - R_1$.
- The flow expands as a linear combination of the form $\Re\left\{\sum_{i=1}^{2} e^{\mathrm{i}\Omega_i T}\left(\mathbf{V}_i, P_i\right)\right\}$

  with $(\mathbf{V}_i, P_i)$ solution of

$$
\begin{aligned}
\nabla \cdot \mathbf{V}_i &= 0, \\
\mathrm{i}\Omega_i \mathbf{V}_i + \frac{1}{\rho}\nabla P_i - \nu \Delta \mathbf{V}_i &= \mathbf{0}, \\
\mathbf{V}_i - \mathrm{i}\Omega_i Q_i \mathbf{W}_i &= \mathbf{0} \quad \text{on} \quad \partial\mathcal{C}_i, \\
\mathbf{V}_i &= \mathbf{0} \quad \text{on} \quad \partial\overline{\mathcal{C}_i}, \\
P_i &= 0 \quad \text{at} \quad X = 0 \ \text{ and } \ X = L.
\end{aligned}
$$

- The fluid force on $\mathcal{C}_i$ due to the motion of $\mathcal{C}_j$ is $\Re\left\{\sum_{j=1}^{2} e^{\mathrm{i}\Omega_j T}\mathbf{F}_{ij}\right\}$ with

$$
\mathbf{F}_{ij} = -\int_{\partial\mathcal{C}_i} P_j \mathbf{n}_i dS_i + \rho\nu \int_{\partial\mathcal{C}_i} \left[\nabla \mathbf{V}_j + \left(\nabla \mathbf{V}_j\right)^T\right] \cdot \mathbf{n}_i dS_i.
$$

Governing equations

Dimensionless Euler equations and boundary conditions write

$$
\begin{aligned}
\nabla^* \cdot \mathbf{v}_i &= 0, \\
\mathrm{i}\mathbf{v}_i + \nabla^* p_i - \frac{1}{Sk_i}\Delta^* \mathbf{v}_i &= \mathbf{0}, \\
\mathbf{v}_i &= \mathrm{i} w_i \mathbf{e}_y \quad \text{on} \quad \partial\mathcal{C}_i, \\
\mathbf{v}_i &= \mathbf{0} \quad \text{on} \quad \overline{\partial\mathcal{C}_i}, \\
p_i &= 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = 1.
\end{aligned}
$$

- $x = X/L$, $t = T\Omega$, $k_i = K_i L$,
- $\mathbf{V}_i = Q_i \Omega_i \mathbf{v}_i$, $P_i = \rho\left(R_2 - R_1\right) Q_i {\Omega_i}^2 p_i$, $\mathbf{F}_{ij} = \rho \pi R_i L R_j Q_j {\Omega_j}^2 \mathbf{f}_{ij}$,
- $\mathbf{W}_i\left(X\right) = w_i \left(\dfrac{X}{L}\right) \mathbf{e}_y$,
- $\nabla^* = \left(R_2 - R_1\right) \nabla$ and $\Delta^* = \left(R_2 - R_1\right)^2 \Delta$,
- Aspect ratio $l = \dfrac{L}{R_1}$, radius ratio $\varepsilon = \dfrac{R_2}{R_1}$, Stokes number $Sk = \dfrac{(R_2 - R_1)^2 \Omega_i}{\nu}$.

New theoretical formulation

New theoretical formulation

- Helmholtz decomposition

$$\mathbf{v}_i = \nabla^* \phi_i + \nabla^* \times \boldsymbol{\psi}_i.$$

- Dimensionless Euler equations yields

$$\Delta^* \phi_i = 0,$$
$$\nabla^* \times \left(\Delta^* \boldsymbol{\psi}_i - \mathrm{i} S k_i \boldsymbol{\psi}_i\right) - S k_i \nabla^* \left(\mathrm{i}\phi_i + p_i\right) = \mathbf{0}.$$

- Taking the divergence and the curl yields

$$p_i = -\mathrm{i}\phi_i,$$
$$\Delta^* \boldsymbol{\psi}_i + {\beta_i}^2 \boldsymbol{\psi}_i = \mathbf{0} \qquad \text{with} \qquad \beta_i = \sqrt{-\mathrm{i} S k_i}.$$

- Dimensionless cylindrical coordinates (r, θ, x) and the associated physical basis $\mathcal{B} = (\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_x)$.
- The boudary condition on $\partial \mathcal{C}_i$, i.e. $\nabla^* \phi_i + \nabla^* \times \boldsymbol{\psi}_i = \mathrm{i} w_i(x) cos(\theta)$ at $r = r_i$, suggests that ϕ_i and $\boldsymbol{\psi}_i$ shall be linear combinations of w_i and dw_i/dx.

New theoretical formulation

- To fulfill the pressure boundary conditions, $p_i(0) = p_i(1) = 0$, w_i is extend to an odd function $\tilde{w}_i$ of period 2.
- The Fourier series of $\tilde{w}_i$ vanishes at $x = 0$ and $x = 1$ and

$$\forall x \in]0,1[\,, \qquad w_i\left(x\right) = \tilde{w}_i\left(x\right) = \sum_{n=1}^{\infty} w_{in}\left(x\right),$$

with $w_{in} = b_{in}\sin\left(n\pi x\right)$ and $b_{in} = 2\int_0^1 w_i\left(x\right)\sin\left(n\pi x\right)dx$ the n-th Fourier coefficient.

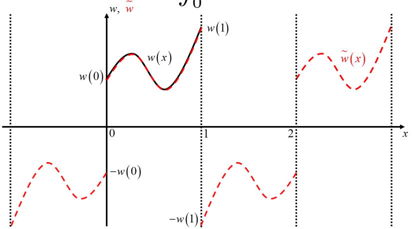


FIGURE – Principle of the extension of w (black solid line) to an odd function $\tilde{w}$ (red dashed line) of period 2.

New theoretical formulation

- We seek fluid functions of the form (linear combinations of the Fourier harmonics of the vibration modes w_{in})

$$\left(\phi_i, \boldsymbol{\psi}_i\right) = \sum_{n=1}^{\infty} \left(\phi_{in}, \boldsymbol{\psi}_{in}\right),$$

$$\phi_{in}(r,\theta,x) = \Phi_{in}(r)\cos(\theta)\, w_{in}(x),$$

$$\boldsymbol{\psi}_{in}(r,\theta,x) = \begin{pmatrix} 0 \\ 0 \\ \Psi_{in}(r)\sin(\theta) \end{pmatrix} w_{in}(x) + \frac{\varepsilon - 1}{l} \begin{pmatrix} \mathrm{A}_{in}(r)\sin(\theta) \\ \mathrm{A}_{in}(r)\cos(\theta) \\ 0 \end{pmatrix} \frac{dw_{in}}{dx}(x),$$

- $\Phi_{in}(r)$, $\Psi_{in}(r)$, and $\mathrm{A}_{in}(r)$ are linear combinations of Hankel functions of the first and second kind.
- Small displacement and deformation of the cylinders, i.e. $\mathbf{n}_i \approx (-1)^{i+1}\, \mathbf{e}_r$.
- The linear dimensionless fluid force is aligned with the direction of the motion.

New theoretical formulation

⇒ The linear dimensionless fluid force writes $\boldsymbol{f}_{ij} = f_{ij}\mathbf{e}_y$ with

$$f_{ij}\left(\varepsilon, l, Sk_j, w_j\right) = (-1)^{i+1}\left(\varepsilon - 1\right)\varepsilon^{1-j}\sum_{n=1}^{\infty}\mathrm{i}\left[\Phi_{jn}\left(r_i\right) + \frac{{\gamma_n}^2\Phi_{jn}\left(r_i\right) - {\alpha_{jn}}^2\Psi_{jn}\left(r_i\right)}{{\gamma_n}^2 - {\alpha_{jn}}^2}\right]w_{jn}.$$

⇒ The dimensionless modal added mass and damping coefficients

$$C_m^{(ij)} - \mathrm{i}C_v^{(ij)} = \langle w_j, f_{ij}\rangle.$$

- Full analytical expressions for the added mass and damping coefficients in still fluid.
- Depends on the radius ratio ε, the aspect ratio l, the Stokes number, Sk, and the imposed vibration mode w.
- Applies for all classical boundary conditions of an Euler-Bernoulli beam.

R. Lagrange and M. A. Puscas
Viscous theory for the vibrations of coaxial cylinders. Analytical formulas for the fluid forces and the modal added coefficients.
J. Appl. Mech, 2023

First mode of a clamped-free beam

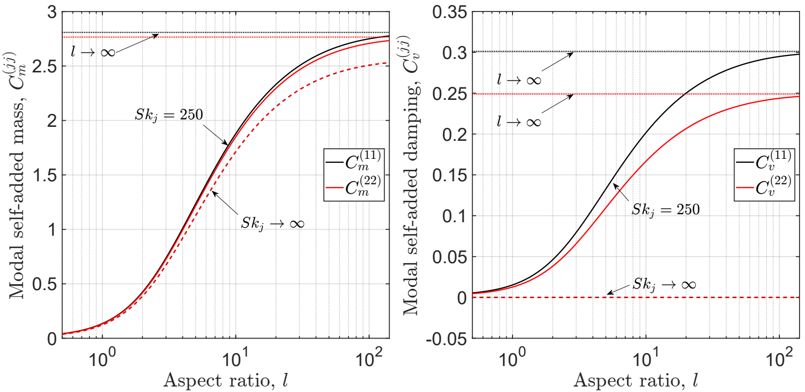


FIGURE – Modal self-added mass and damping coefficients as functions of the aspect ratio, l, for $i = j = 1$ (black) and $i = j = 2$ (red). The cylinder $\mathcal{C}_j$ vibrates in the first mode of a clamped-free beam. The solid lines correspond to a Stokes number $Sk_j = 250$. The dashed lines (black and red are indistinguishable) correspond to the inviscid limit $Sk_j \to \infty$. The dotted lines correspond to the limit of infinitely long cylinders, $l \to \infty$, for $Sk_j = 250$. The radius ratio is $\varepsilon = 1.1$.

Theory vs. Numerics

Clamped-free cylinders vibrating in the first mode

- Incompressible turbulent fluid solver : **open source TrioCFD** software.
- The FSI problem involving moving boundaries is solved using an Arbitrary Lagrange-Eulerian method (ALE).
- Imposed a displacement of the form $Q \sin(\Omega_j T) w(X/L_j)$ with amplitude $Q = 5 \cdot 10^{-5}$ [uol] (units of length) and a forcing frequency $\Omega_j/(2\pi) = 10$ [uof] (units of frequency).
- First mode clamped-free vibration : $w\left(x\right) = \cosh\left(\lambda x\right) - \cos\left(\lambda x\right) - \sigma \sinh\left(\lambda x\right) + \sigma \sin\left(\lambda x\right)$, $\lambda = 1.875$ and $\sigma = 0.734$.
- The inner radius is $R_1 = 0.02$ [uol] and the radius of the outer cylinder is $R_2 = 0.022$ [uol].
- The Stokes number is $Sk_j = 250$.

Clamped-free cylinders vibrating in the first mode

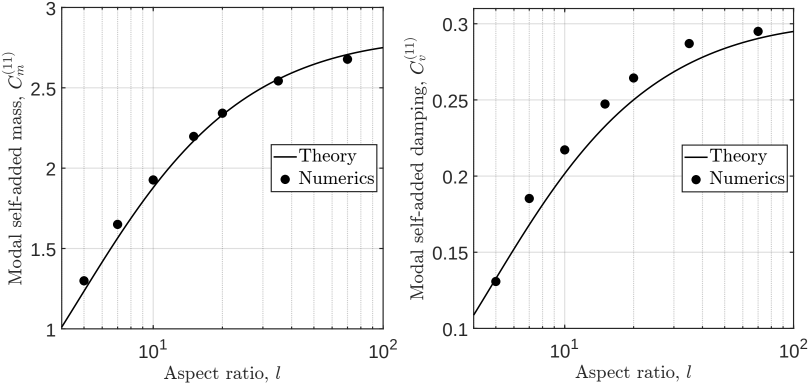


FIGURE – Modal self-added mass and damping coefficients as functions of the aspect ratio, l. The cylinder $\mathcal{C}_1$ vibrates in the first mode of a clamped-free beam. The Stokes number is $Sk_1 = 250$. The radius ratio is $\varepsilon = 1.1$.

Clamped-sliding cylinders

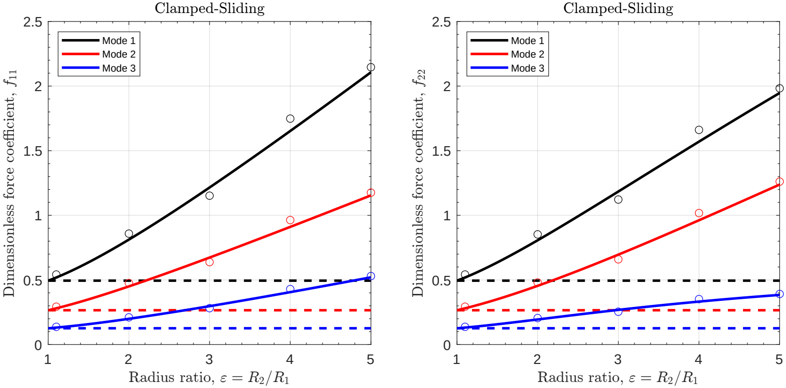


FIGURE – Clamped-sliding case. Evolution of the dimensionless force coefficients, f_{ii} as a function of the radius ratio, $\varepsilon = R_2/R_1$. The cylinder $\mathcal{C}_i$ vibrates in the three first modes of a clamped-sliding beam. Continuous lines correspond to the new theoretical prediction. Dashed lines correspond to the narrow gap prediction [Lagrange & Puscas, JFS, 2022]. Open circles correspond to numerical simulations. The aspect ratio is $l = 35$.

Conclusions and perspectives

Conclusions :

- New theoretical formulation to estimate the modal fluid force and the added mass and damping coefficients.
- Based on a Helmholtz expansion of the fluid velocity.
- Includes the viscous effects of the fluid so that added-damping is accounted.
- It applies to all sizes of the fluid gap.
- Accounts for the finite length of the cylinder.
- Covers all type of classical forced beam vibrations boundary conditions.
- Good agreement between the numerical simulations and the new theoretical formulation.

Perspectives :

- Simulate and model the free oscillations of a slender confined structure in a still fluid.
- Carry out parametric studies to determine new dimensionless scaling laws for frequency and damping.
- Derive analytical developments to include the effect of axial fluid flow on the added-coefficients.
- Determine linear stability properties of the coupled system.

Thank you for your attention