Some new results on the numerical simulation of thick sprays

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Sprays describe particles in interaction with a surrounding gas. We are interested in the so-called "thick" sprays with a coupling through a drag force and the volume fraction of the gas [1, 2]. Such model usually takes the form of a coupling between a kinetic equation such as a Vlasov equation, and a system laws of conservation such as the Euler equations

$$\begin{cases} \partial_t(\alpha\varrho) + \boldsymbol{\nabla}\cdot(\alpha\varrho\boldsymbol{u}) = 0 \\ \partial_t(\alpha\varrho\boldsymbol{u}) + \boldsymbol{\nabla}\cdot(\alpha\varrho\boldsymbol{u}\otimes\boldsymbol{u}) + \boldsymbol{\nabla}p = m_\star\boldsymbol{\nabla}p\displaystyle\int_{\mathbf{R}^3} f\,\mathrm{d}v + D_\star\int_{\mathbf{R}^3}(\boldsymbol{v}-\boldsymbol{u})f\,\mathrm{d}v \\ \partial_t(\alpha\varrho e) + \boldsymbol{\nabla}\cdot(\alpha\varrho e\boldsymbol{u}) + p(\partial_t\alpha + \boldsymbol{\nabla}\cdot(\alpha\boldsymbol{u})) = D_\star\displaystyle\int_{\mathbf{R}^3}|\boldsymbol{v}-\boldsymbol{u}|^2 f\,\mathrm{d}v \\ \partial_t f + \boldsymbol{v}\cdot\boldsymbol{\nabla}_x f + \boldsymbol{\nabla}_v\cdot(\Gamma f) = 0 \\ \alpha = 1 - m_\star\displaystyle\int_{\mathbf{R}^3} f\,\mathrm{d}v \\ m_\star\Gamma = -m_\star\boldsymbol{\nabla}p - D_\star(\boldsymbol{v}-\boldsymbol{u}) \end{cases}$$

The numerical simulation of such model is a difficult problem, especially in the regime where the volume fraction is close to zero. We propose a scheme of Finite Volume type for the fluid part, and a semi-Lagrangian scheme for the Vlasov part, which is conservative in mass and total momentum. We will discuss the problem of the positivity of the volume fraction, and we will discuss a method to ensure this positivity, using ideas used by Bertrand Maury and his collaborators in the context of crowd motion [3].

[1] L. Boudin, L. Desvillettes, R. Motte. *A modeling of compressible droplets in a fluid.* Commun. Math. Sci., **1(4)**, 657–669, 2003.

[2] C. Buet, B. Després, L. Desvillettes. *Linear stability of thick sprays equations.* Journal of Statistical Physics, **190(53)**, 2023. doi :10.1007/s10955-022-03057-4.

[3] B. Maury, A. Roudneff-Chupin, F. Santambrogio, J. Venel. *Handling congestion in crowd motion modeling.* Networks and Heterogeneous Media, **6(3)**, 485–519, 2011. doi :10.3934/nhm.2011.6.485.

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