Linearized Vlalsov-Maxwell system and plasma heating by the eXtraordinary mode

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In this talk, we present a model of plasma heating (that is a configuration of ions where the heating of the plasma is driven by the electric field, in the framework of an imposed vertical magnetic field B_0). The ions density of the plasma is $n_0(x)$, the given configuration of the plasma is characterized by $n_0(x)F^0(v)$ (Maxwellian distribution).
The model studied is

$$\begin{array}{l}\partial_t f + v.\nabla_x f + \frac{qB_0}{m}(\frac{v}{c}\wedge e_3).\nabla_v f + \nu f = -\frac{qn_0(x)}{m}(E + \frac{v}{c\mu_0}\wedge H).\nabla_v F_0(v)\\ \mu_0\partial_t H + \nabla_x \wedge E = 0\\ \epsilon_0\partial_t E - \nabla_x \wedge H = j\\ j(t,x) = \frac{qn_0(x)}{m}\int v f(t,x,v)dv,\end{array}$$

where $\nu > 0$ is a dissipation constant.
A traditional approach is to replace the Vlasov equation by the cold plasma model equation for the current $\partial_t j + \frac{qB_0}{m}(j\wedge e_3) + \tilde{\nu} j = \frac{q^2 n_0(x)}{m}E$, we will recall how to derive the cold plasma model equation from the Vlasov equation under the hypotheses $\int vF_0(v)dv = 0$ (particles at rest) and k_bT small (cold plasma). We also mention how to derive f (or rather $\partial_t f$) in general, showing an existence and uniqueness result (without Cauchy initial condition but imposing conditions on the solution f).
We then study solutions of the cold plasma model after Fourier transform in (t,y,z) and resume to a system of four ODEs with coefficients depending on x, for which we identify the singularities (regular-singular points). When $\tilde{\nu}\to 0_+$, the singularities of the system (response of the plasma to an imposed frequency in time ω) are at the points x such that

$$(\frac{qB_0}{m})^2 + \frac{q^2 n_0(x)}{m\epsilon_0} = \omega^2. \quad (1)$$

These resonances are called the hybrid singularities because they neither occur at the plasma frequency $\sqrt{\frac{q^2 n_0(x)}{m\epsilon_0}}$ nor at the cyclotron frequency $|\frac{qB_0}{m}|$.
The model ODE for this problem amounts to be a Bessel-type ODE and one has

Théorème 1. *The heating of the plasma occurs at points x_h such that (1) is satisfied, the amplification of the solution of the cold plasma model system when $\tilde{\nu}\to 0_+$ occurs at these points, and one has E_2, $ik_3H_2 - ik_2H_3$ behave as $\ln|x-x_h|$, and $E_1 \simeq \frac{c}{x-x_h+iO(\tilde{\nu})}$ when $\tilde{\nu}\to 0_+$.*

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