How effective is optical flow for collision avoidance in drone swarms ?

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Context : drone swarms

- Today we use a few drones :
  - ▷ Mission : check structure state, aerial ballets ...
  - ▷ Piloted or preprogrammed

- Tomorrow, there will be several thousands :
  - ▷ Mission : surveillance, search ...
  - ▷ Too many parameters for planification

⇒ Autonomy needed with simple control laws

Modeling

- My thesis : Large population dynamics description
  - ▷ Autonomous and interchangeable agents
  - ▷ Tasks to complete
  - ▷ Fast numerical simulations

- Command laws computed from information gathered by agents
  - ▷ Simple and generic
  - ▷ Constraints : no collision, no swarm dispersion ...
  - ▷ Simple individual behaviors $\longrightarrow$ Group behavior

For this talk, we are interested in a command law to prevent collisions

Intuitive modeling

Microscopic description : Drone i = { position x_i, velocity v_i, command laws }, $i = 1, \cdots, N$

Newtonian model

$$\begin{cases} x_i'(t) = v_i(t) \\ m v_i'(t) = F_i^{rep} + F_i^{att} + F_i^{mim} + \dots \end{cases}, i = 1, \cdots, N.$$

Behavior	Force
No collision	Repulsion
Swarm cohesion	Attraction
Velocities alignment	Mimetism

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Obstacle	Detection
Target zone	Attraction
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A generic framework for drone mimetism

Cucker-Smale model [1](2007)

$$v_i'(t) = F_i^{mim}(t) = \frac{1}{N}\sum_{j=1}^{N} \psi(|x_i - x_j|)(v_j - v_i)$$

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Features [2](2009) :

- Speed alignment : $\forall i, v_i(t) \underset{t\to\infty}{\to} v_{mean}$
- No pattern formation
- ~Collision avoidance : may not be guaranteed, depends on ψ and the initial conditions

Optical flow

- Measures the displacement of the velocity field, used by bees for instance

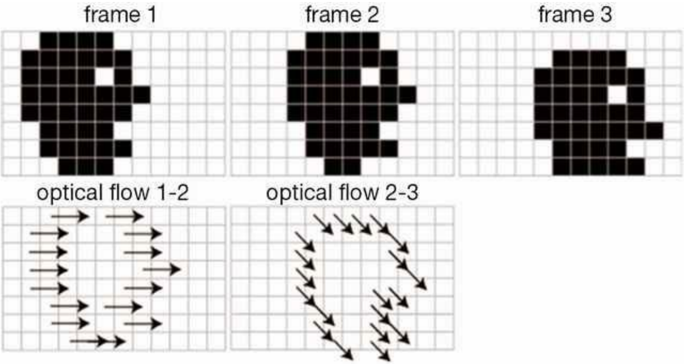

Figure – Illustration of optical flow, taken from [3](2014)

Optical flow in drones

- Cameras on drones capture optical flow
- We reconstruct the relative velocity between the camera and objects [4](2012)
  - ▷ BUT weighted by their relative distance !

$$w_{ij} = \frac{v_i - v_j}{|x_i - x_j|}$$

- We build a command law from this input, by taking the average over all drones :

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- This is a singular Cucker-Smale model ! $\psi(r) = \frac{1}{r}$

Optical flow for collision avoidance

Known results, Cucker-Smale [1](2007), Carrillo-Choi-Mucha-Peszek [5](2016)

Mimetism :

$$\text{If } \int_1^\infty \psi(r)dr = \infty, \text{ then } \forall i, v_i(t) \xrightarrow[t\to\infty]{\text{exp.}} \frac{1}{N}\sum_{j=1}^N v_j(0),$$

No collision :

$$\text{If } \int_0^1 \psi(r)dr = \infty, \text{ then } \forall T > 0, \exists d_T > 0 : \forall t \le T, \forall i, j, |x_i - x_j|(t) \ge d_T.$$

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- Drones may get infinitely close for long flights ! i.e. $d_T \underset{T\to\infty}{\to} 0$

Optical flow for collision avoidance

Main issue : drones have a (small) size !

- Need to provide a security distance between drones for all time
- Drones no longer are limited to a point representation

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Theorem : M., Dufour, Hérissé (under revision, [6](2022))

If $\psi \in \mathcal{C}^0(\mathbb{R}^+, \mathbb{R}^+)$ and $\int_1^{\infty} \psi(r)dr = \int_0^1 \psi(r)dr = \infty$, then there exists a lower bound on the relative distances that is independent of time, for any non-collisional initial conditions :

$$\exists d > 0 : \forall t \geq 0, \forall i, j, |x_i - x_j|(t) \geq d.$$

Sketch of the proof

- Existence and unicity built upon existing results on the CS models [5](2016)
- Speeds and positions converge exponentially fast in the CS model
- Suppose a contradiction : there is no minimal distance
- Idea : Use the fast convergence to bound ψ which is singular at 0

Main idea

- Split the fleet in two parts : define the set $\mathcal{L}$ of colliding particles with drone l :

$$\mathcal{L} := \{j \in [\![1, N]\!] \mid \exists \{(t_n)\}_{\mathbb{N}} \text{ s.t. } |x_l - x_j|(t_n) \underset{n\to\infty}{\longrightarrow} 0\}, \qquad \mathcal{L}^C = [\![1, N]\!] \setminus \mathcal{L}$$

- Control quantities : $|x|_{\mathcal{L}}(t) := \sqrt{\sum_{i,j\in\mathcal{L}} |x_i - x_j|^2}, \quad |v|_{\mathcal{L}}(t) := \sqrt{\sum_{i,j\in\mathcal{L}} |v_i - v_j|^2},$

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- Colliding particles actually converge
  - ▷ $|x|_{\mathcal{L}} \underset{t\to t_0}{\longrightarrow} 0$ since collision at t_0

- Non-colliding particles are separated from $\mathcal{L}$ after some time

Main estimation

- System of differential equations using the exponential convergence of the model :

$$\begin{cases} \dfrac{d}{dt}|x|_{\mathcal{L}} \leq |v|_{\mathcal{L}}, \\ \dfrac{d}{dt}|v|_{\mathcal{L}} \leq \underbrace{-C_0\psi(|x|_{\mathcal{L}}(t))|v|_{\mathcal{L}}}_{\mathcal{L}\text{ interactions}} + \underbrace{C_1 \exp(-2\psi(r_{max})t)}_{\mathcal{L}^C\text{ interactions}}. \end{cases}$$

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- Therefore

$$\left|\int_T^t \frac{d}{ds}\left(\psi(|x(s)|_{\mathcal{L}})\right) ds\right| \leq \frac{C_1}{2C_0\psi(r_{max})} e^{-2\psi(r_{max})T} + \frac{|v_0|_2}{C_0} < \infty, \qquad \forall t > 0$$

- Contradiction at time of collision : $\mathcal{L}$ has to be empty

$$\psi(r) \underset{r\to 0}{\longrightarrow} \infty \quad \Rightarrow \quad \left|\int_T^t \frac{d}{ds}\left(\psi(|x(s)|_{\mathcal{L}})\right) ds\right| \underset{t\to t_0}{\longrightarrow} \infty.$$

Resistance to uncertainty in measurements

Is the result robust ?

- Errors due to captors precision or delay in information
  - If r_{ij} is unknown, but we can bound the error up to a small multiplicative factor :

$$v_i'(t) = \frac{1}{N}\sum_{j=1}^{N} \frac{v_i - v_j}{\tilde{r_{ij}}}, \quad \tilde{r_{ij}} \in \Big[(1-\varepsilon)|x_i - x_j|, (1+\varepsilon)|x_i - x_j|\Big]$$

- Then the factor is found in the estimation :

$$\left| \int_T^t \frac{d}{ds}\Big(\psi\Big((1+\varepsilon)|x(s)|_{\mathcal{L}}\Big)\Big) ds \right| \leq (1+\varepsilon)\left(\frac{|v_0|_2}{C_0} + \frac{C_1}{2(1+\varepsilon)C_0\psi(r_{max})}\right)$$

Resistance to uncertainty in measurements

- As a result, small ε variations will heavily modify the d_{min} estimation !

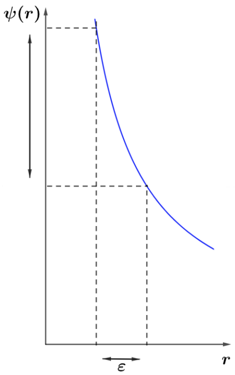

Figure – Illustration of the displacement for a small offset ε in a singular function ψ.

Optical flow for collision avoidance

Is this result robust ?

- Holds for non-complete communication graphs (spanning tree)
- Holds with a trade off between ψ and initial conditions : if only $|v_{init}|_2 \leq \int_{|x_{init}|_2}^{\infty} \psi(s)ds$

▷ Both conditions preserve the exponential convergence

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- ▷ Both conditions preserve the exponential convergence
- Holds adding extra forces ? :

$$v_i'(t) = F_i^{mim}(t) + F_i^{target}(t, v_i) + F_i^{target}(t, x_i) + F_i^{att-rep}(t)$$

$$= \frac{1}{N}\sum_{j=1}^{N} \psi(|x_i - x_j|)(v_j - v_i) + \gamma_v(v_{tar}(t) - v_i) + \gamma_x(x_{tar}(t) - x_i) + \frac{1}{N}\sum_{j=1}^{N} \varphi(|x_i - x_j|)\frac{x_j - x_i}{|x_i - x_j|}$$

keeps exponential alignment, contradicting effect, repulsion injects energy → need estimation

Resistance (?) to extra forces

- Counter example : tracking a target
  - ⇒ Because of constant energy input in the system

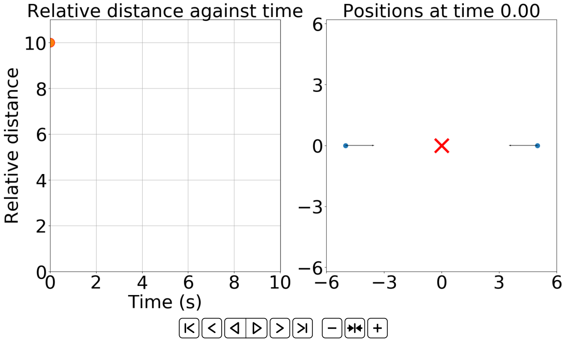

Figure – Initial condition of two drones in 2D subject to optical flow and attracted to a target, and its corresponding minimal distances w.r.t. time.

Resistance (?) to extra forces

- Most interesting scenario : attraction and repulsion forces
  - $\Rightarrow$ They define the formation of the fleet, but bring drones closer

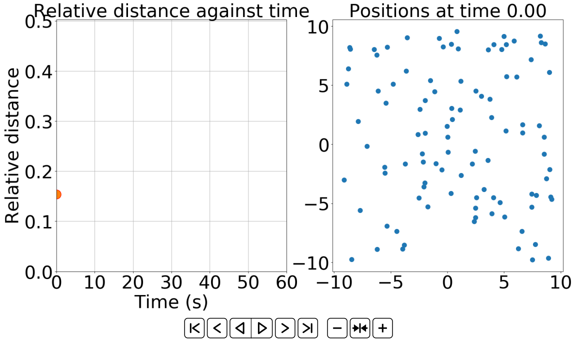

Figure – Evolution of a 100 drones 2D swarm subject to optical flow and an attraction-repulsion law, and its corresponding minimal distances w.r.t. time.

Conclusion

Optical flow for collision avoidance

- Simple command law, passing to the macroscopic limit
- Security distance for a wide range of swarm (speed target, attraction-repulsion, leader...)
- Available for any initial conditions, BUT the minimal distance varies
- Estimation of said minimal distance remains a challenge
- Can it be adapted to more complex models ?

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Thank you for your attention !

Questions ?

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