A HDG+ method for harmonic waves with convection

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This work is dedicated to the memory of Prof. Francisco-Javier Sayas, whose outstanding contributions to HDG methods are summarized in [1].

We consider convected wave equations in the purpose of defining an advanced computational framework for studying the oscillations of the Sun. We made the choice of using the open-source software package `hawen`, see [2] and `hawen`'s web page, which is based upon Hybridizable Discontinuous Galerkin method. HDG formulations allow, via static condensation, to considerably reduce the computational costs of DG methods which can be a hindrance to the solution of large scale wave equations.

As a first step, we have constructed three variations of the Hybridizable Discontinuous Galerkin (HDG) method to solve the convected Helmholtz equation, see [3, 4]. HDG methods are mixed DG methods whose solution is computed thanks to a surfacic auxiliary unknown. This allows for an efficient implementation of the method that relies on a static condensation process, leading to a so-called global problem for this skeleton unknown only. The original unknowns can then be reconstructed locally in a parallel way.

In this talk, we focus on the so-called HDG+ method, see [5, 6], which has two distinctive features among the family of HDG methods : the local fields are approximated by polynomials of different degrees, and a reduced stabilization technique is used to reduce the size of the global problem. As a result of this, HDG methods keep the advantages of the DG methods (eg. hp-adaptivity, high order,...) without incurring their high numerical cost. We present theoretical results including well-posedness of the local and global problems, convergence analysis for regular solutions and a discussion about the choice of penalization parameters. To illustrate the theoretical results, we then provide implementation details and numerical results for both academic and realistic cases using the `hawen` solver. The performance of the constructed HDG+ method is then compared to standard finite elements and other DG methods.

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[2] Florian Faucher. ‘hawen‘ : Time-harmonic wave modeling and inversion using hybridizable discontinuous Galerkin discretization. *Journal of Open Source Software*, 6(57) :2699, January 2021.

[3] Hélène Barucq, Nathan Rouxelin, and Sébastien Tordeux. HDG and HDG+ methods for harmonic wave problems with convection. *Rapport de recherche Inria 9410*, 2021.

[4] Hélène Barucq, Nathan Rouxelin, and Sébastien Tordeux. Construction and analysis of a HDG solution for the total-flux formulation of the convected Helmholtz equation. *Mathematics of Computation*, 2023.

[5] Cristpoh Lehrenfeld. *Hybrid Discontinuous Galerkin methods for solving incompressible flow problems.* PhD thesis, RWTH Aachen, 2010.

[6] Shukai Du and Francisco-Javier Sayas. *An Invitation to the Theory of the Hybridizable Discontinuous Galerkin Method.* Springer International Publishing, 2019.

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