# OPTIMAL CONTROL OF SOLAR SAILS

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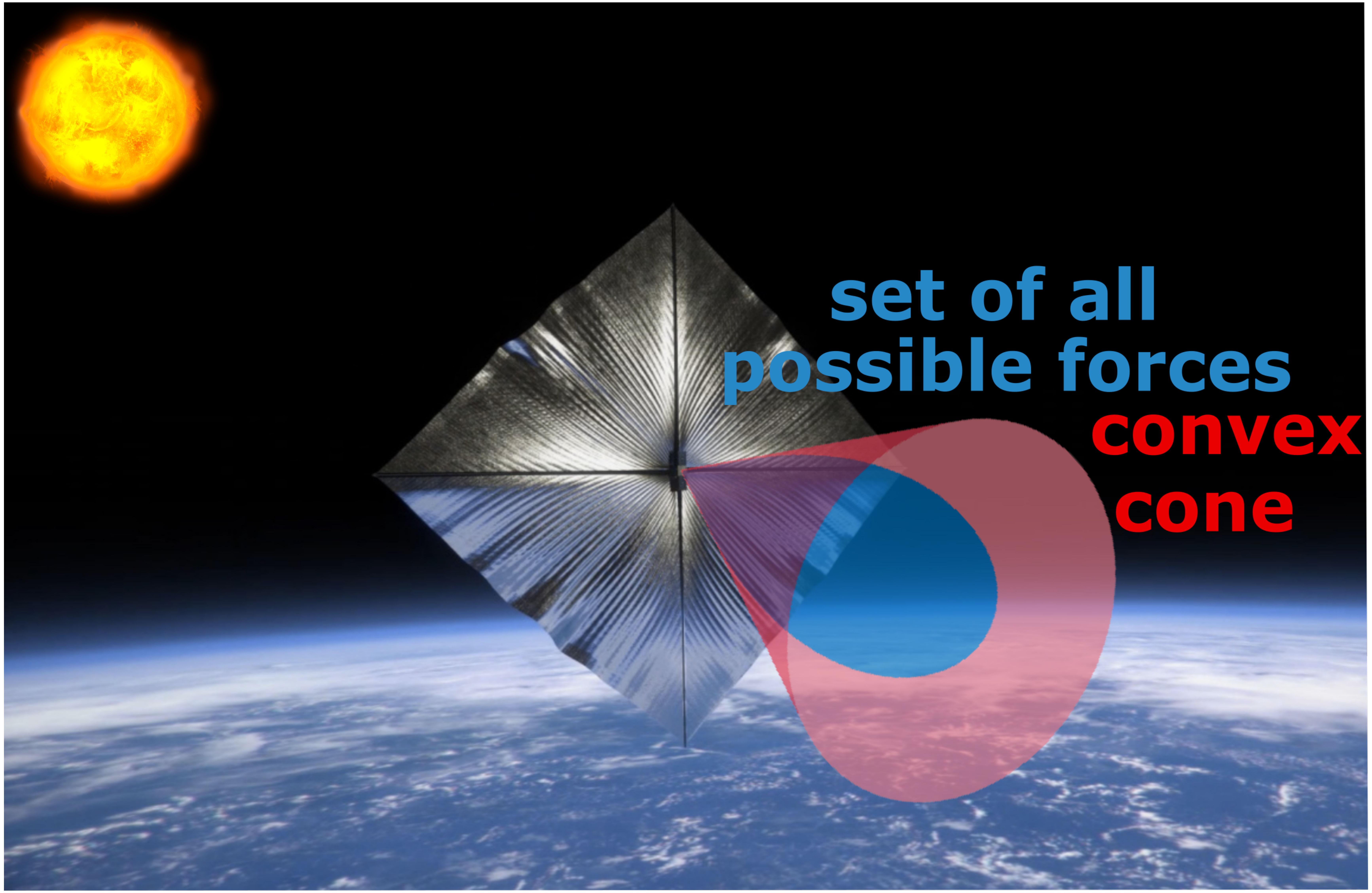
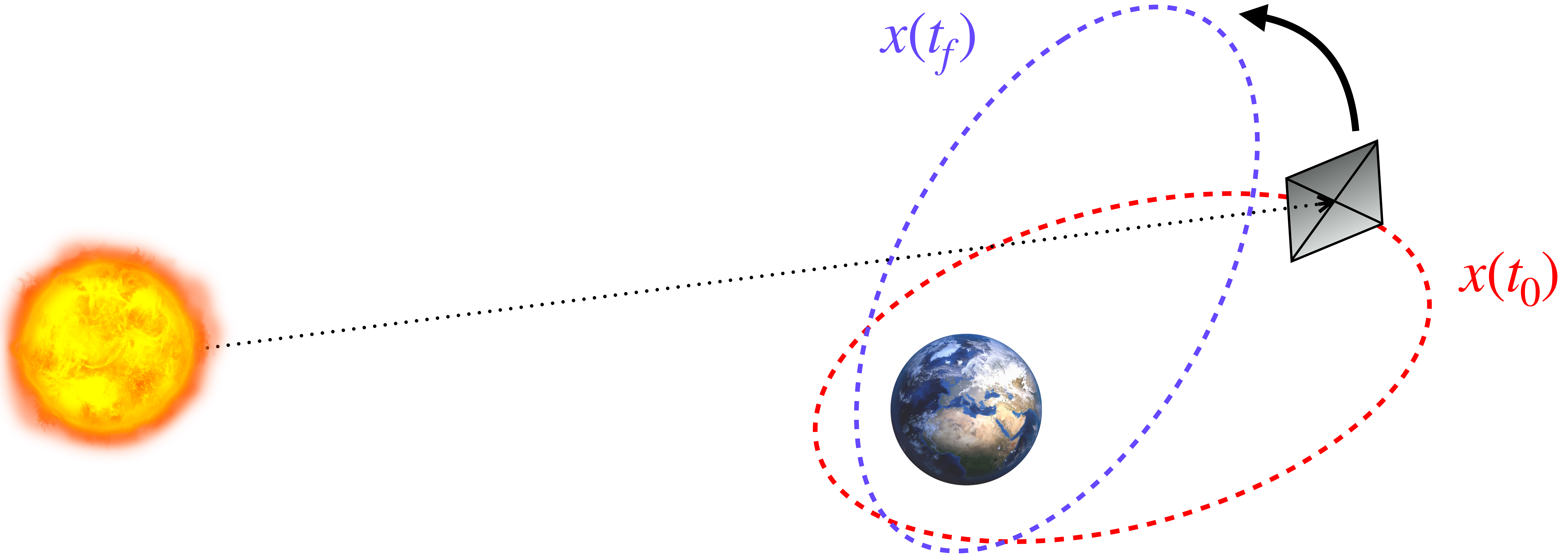
# Non-ideal sail: a cone-constrained control problem

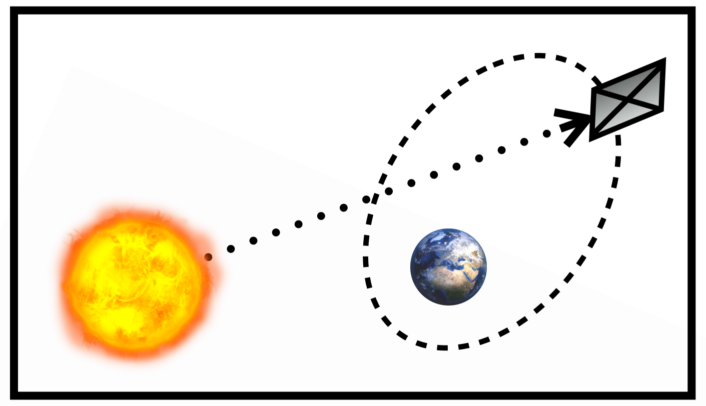


# How to optimally change an orbit of a non-ideal sail?




How to generate any $x(t_0) \longrightarrow x(t_f)$ ?

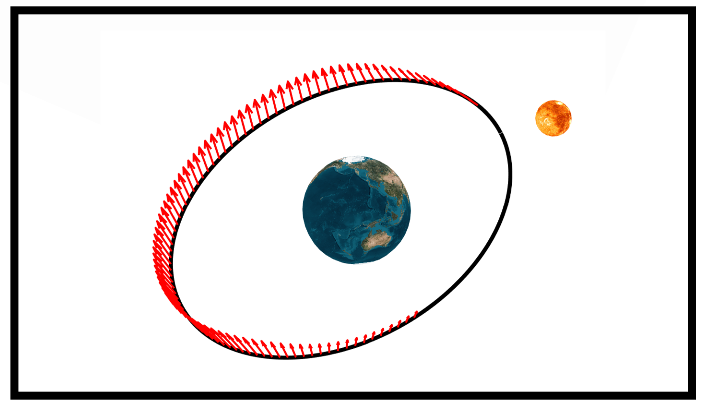
# Outline



1. Dynamics of the system

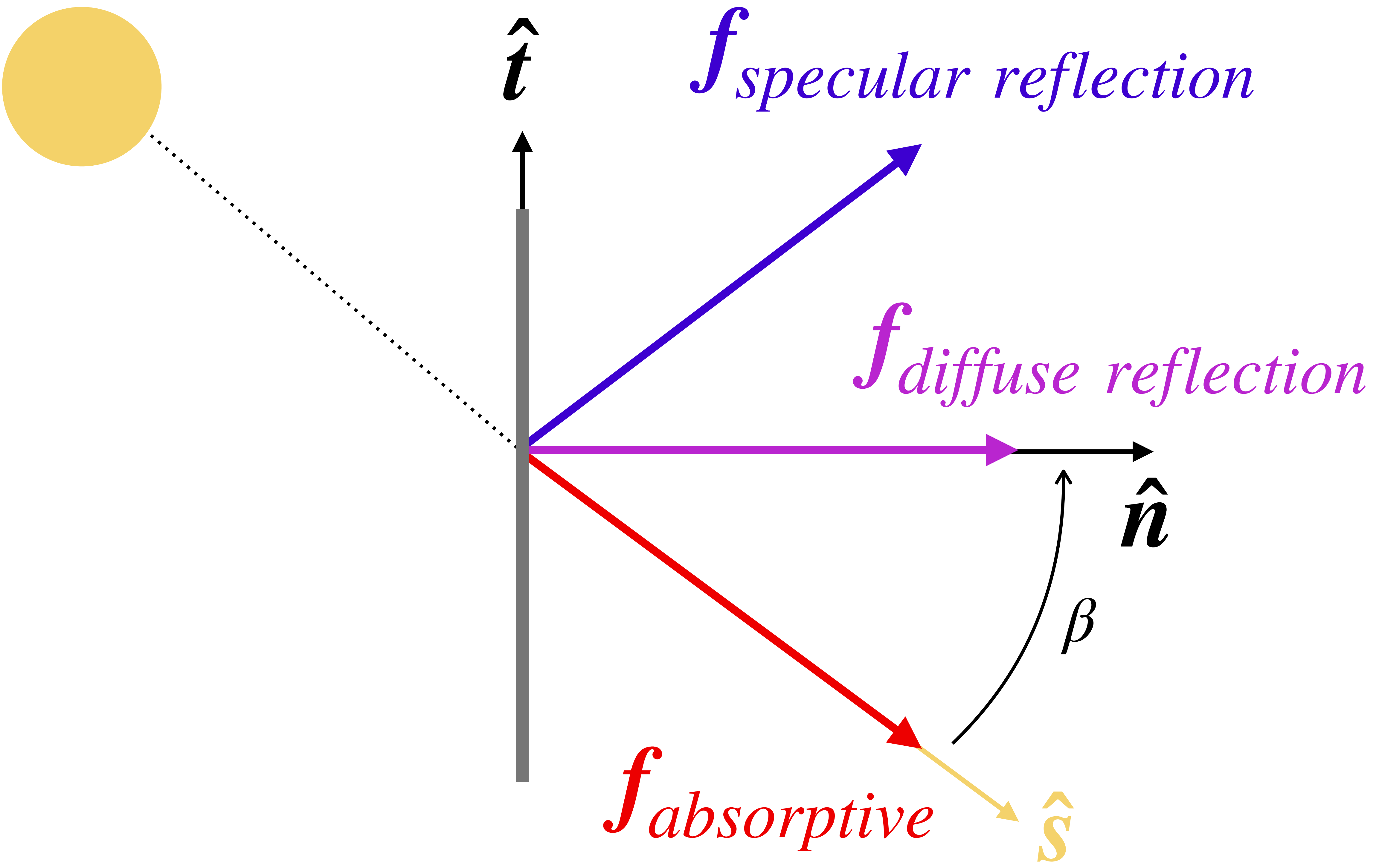
$$\max_{u\in U} H$$

2. Necessary conditions for optimality



3. Algorithm for optimal control of solar sails

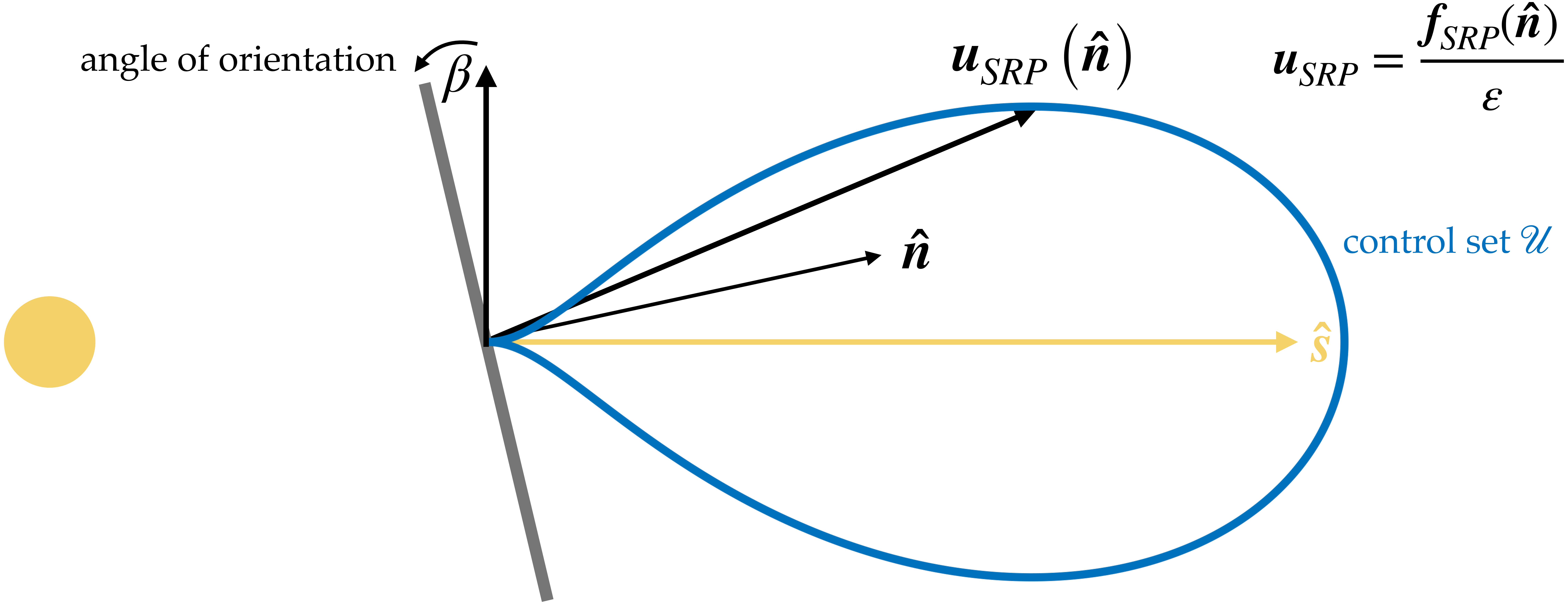
# 1. Force components of solar sail



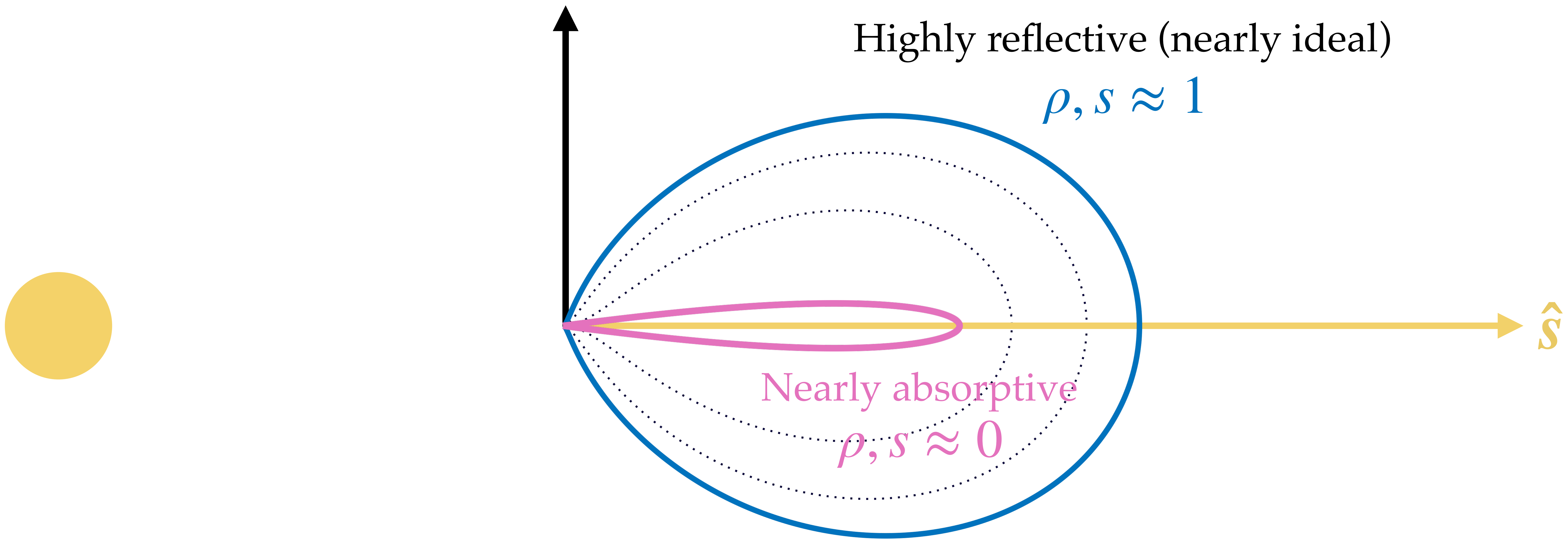

$$f_{SRP} = f_{absorptive} + f_{specular\ reflection} + f_{diffuse\ reflection}$$

# 1. Control set

$$\dot{x} = F^0(x) + \varepsilon \sum_i u_i F^i(x), \quad u \in \mathscr{U}$$

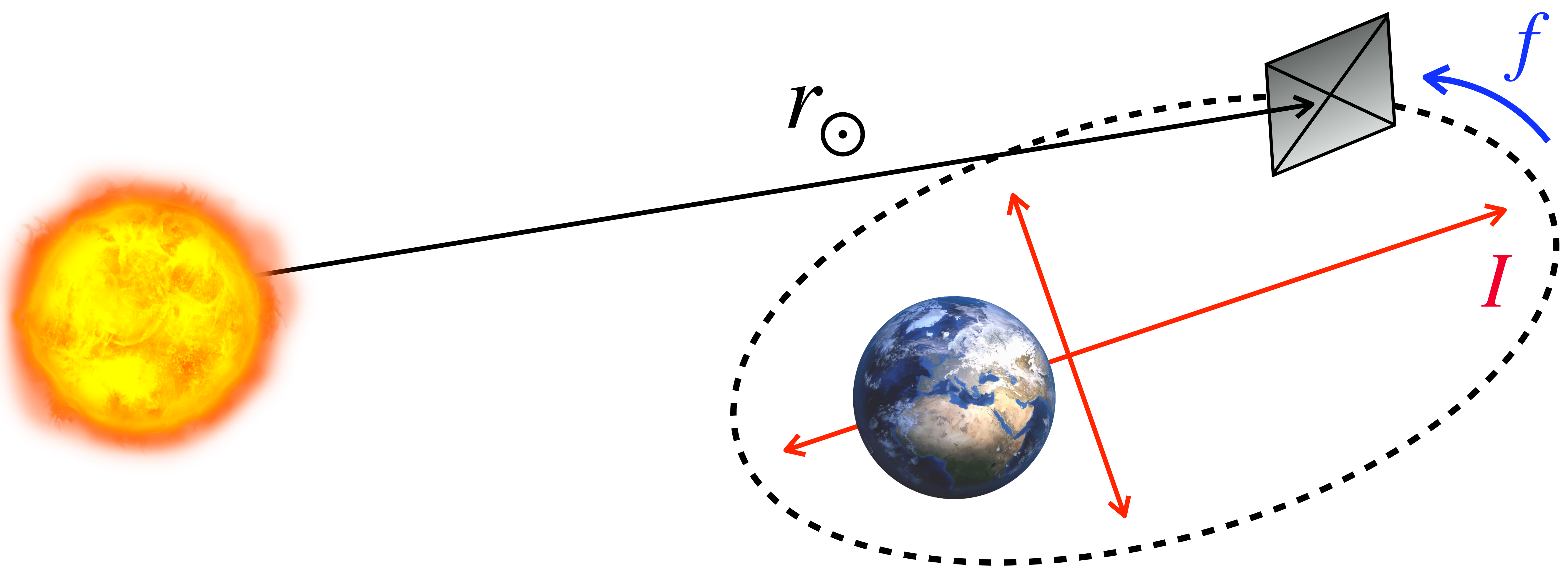


# 1. Parametrization of the control set




$\rho, s \in [0,1]$ portion of specular, diffuse reflection

# 1. Dynamical system




**Assumptions**:

No eclipses

Sun motion neglected over one orbit

SRP is the only perturbation

$$\dot{x} = F^0(x) + \varepsilon \sum_i u_i F^i(x), \quad u \in \mathcal{U}, \quad i = 1, 2, 3$$

with $x = (I, f)$, $\quad I \in M$, $f \in \mathbb{S}^1$, $\quad F^0, F^i$ given by Gauss variational equations

# 2. Optimal control problem

$$\frac{\mathrm{d}\delta I}{\mathrm{d}f} = \varepsilon\, F(\bar{I}, f)\, u$$

$$\max_{u(f)\in\mathcal{U}} \delta I(2\pi)\cdot d_I \qquad \text{subject to} \qquad \begin{aligned} &\delta I(2\pi) = \varepsilon \int_{\mathbb{S}^1} \sum_{i=1}^{3} u_i\, F_i(\bar{I}, f)\, \mathrm{d}f \\ &\delta I(0) = 0, \qquad \delta I(2\pi) \parallel d_I \end{aligned}$$

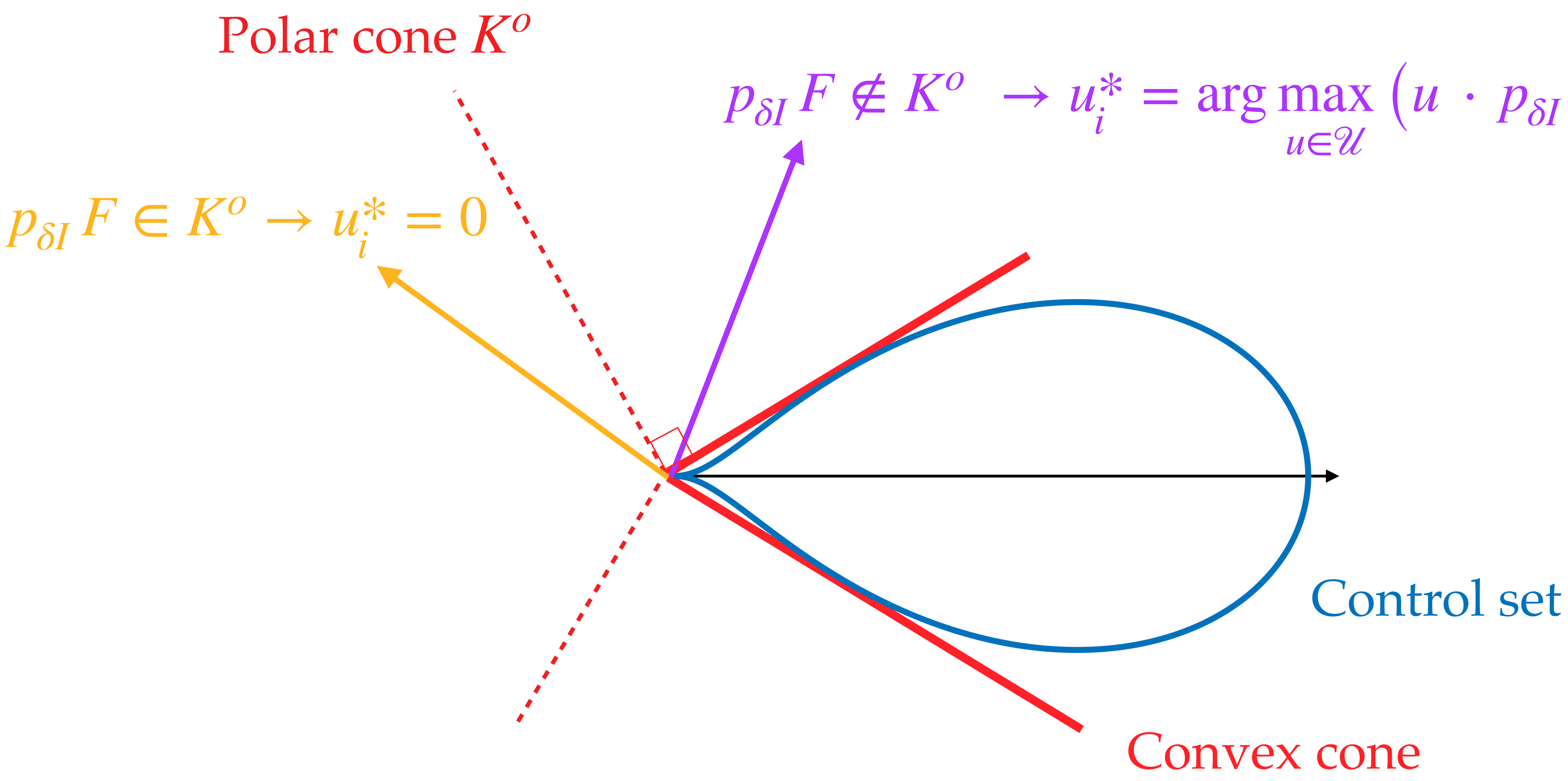
# 2. Necessary conditions for optimality

$$H(\bar{I}, f, p_{\delta I}, u) = \varepsilon \, u \, p_{\delta I} \, F(\bar{I}, f)$$

$$u^* = \arg\max_{u \in \mathcal{U}} H = \arg\max_{u \in \mathcal{U}} \left( u \cdot p_{\delta I} \, F(\bar{I}, f) \right)$$

# 2. Geometrical interpretation of PMP

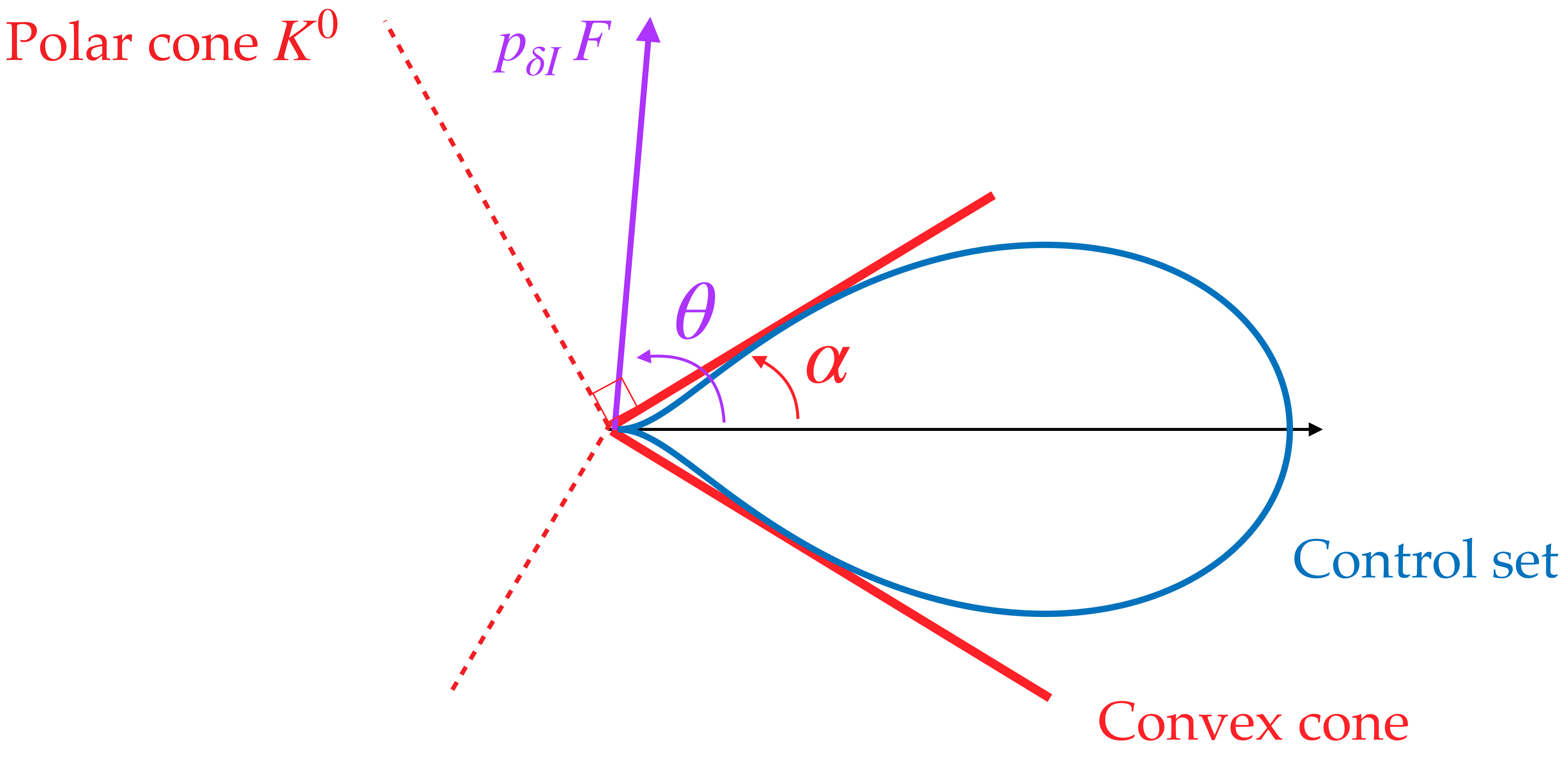
$$u^* = \arg\max_{u \in \mathcal{U}} \left(u \cdot p_{\delta I} F(\bar{I}, f)\right)$$



# 2. What is the structure of the solution?

Shooting variable: $p_{\delta I}$

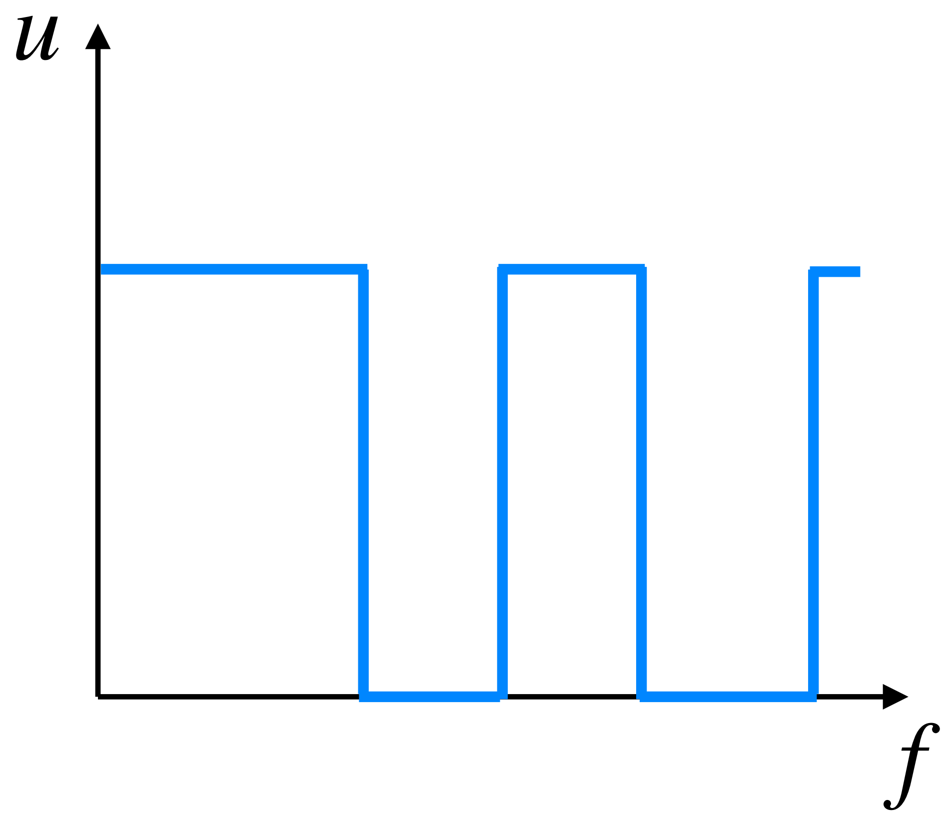
Switches between bangs and zeros: $p_{\delta I}\, F(\bar{I}, f) \in \partial K^o \iff \theta = \alpha + \dfrac{\pi}{2}$



# 2. What is the structure of the solution?

Shooting variable: $p_{\delta I}$

Switches between bangs and zeros: $\underbrace{p_{\delta I}}_{\text{constant}} \underbrace{F(\bar{I}, f)}_{\text{trigonometric polynomial of degree 4}} \in \partial K^0 \iff \theta = \alpha + \frac{\pi}{2}$




max number of roots = **8** $\iff$ max number of bangs = **5**

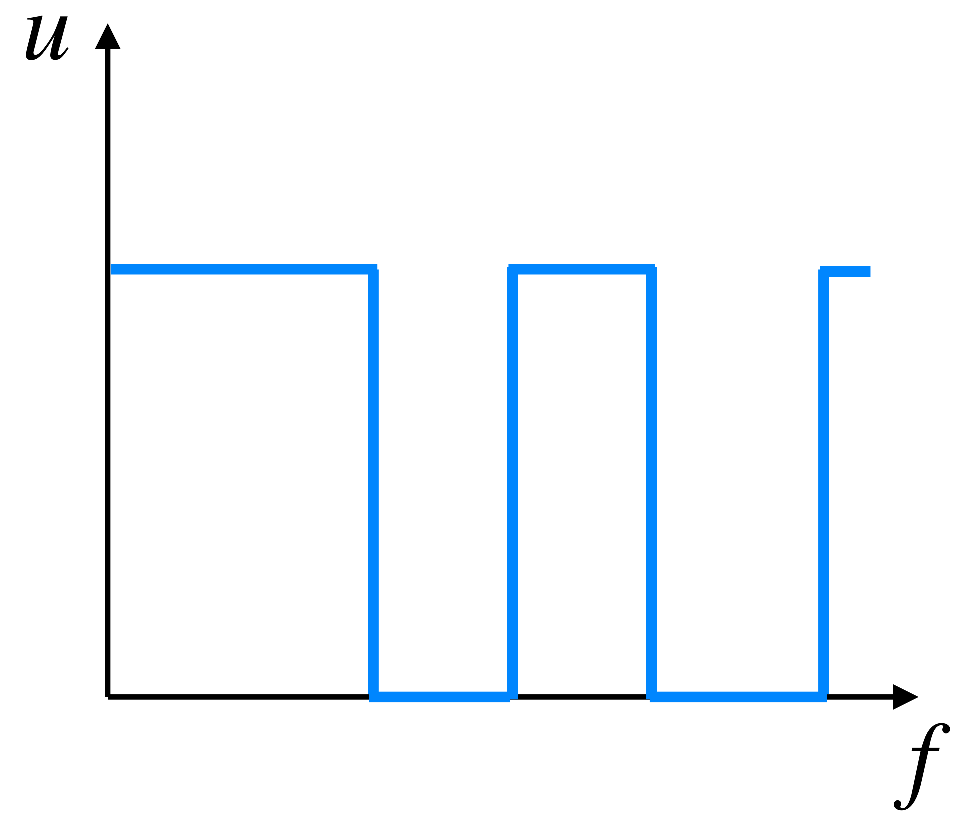
# 2. What is the structure of the solution?

Shooting variable: $p_{\delta I}$ Initial guess ?

Switches between bangs and zeros: $p_{\delta I} \, F(\bar{I}, f) \in \partial K^0 \iff \theta = \alpha + \dfrac{\pi}{2}$

constant

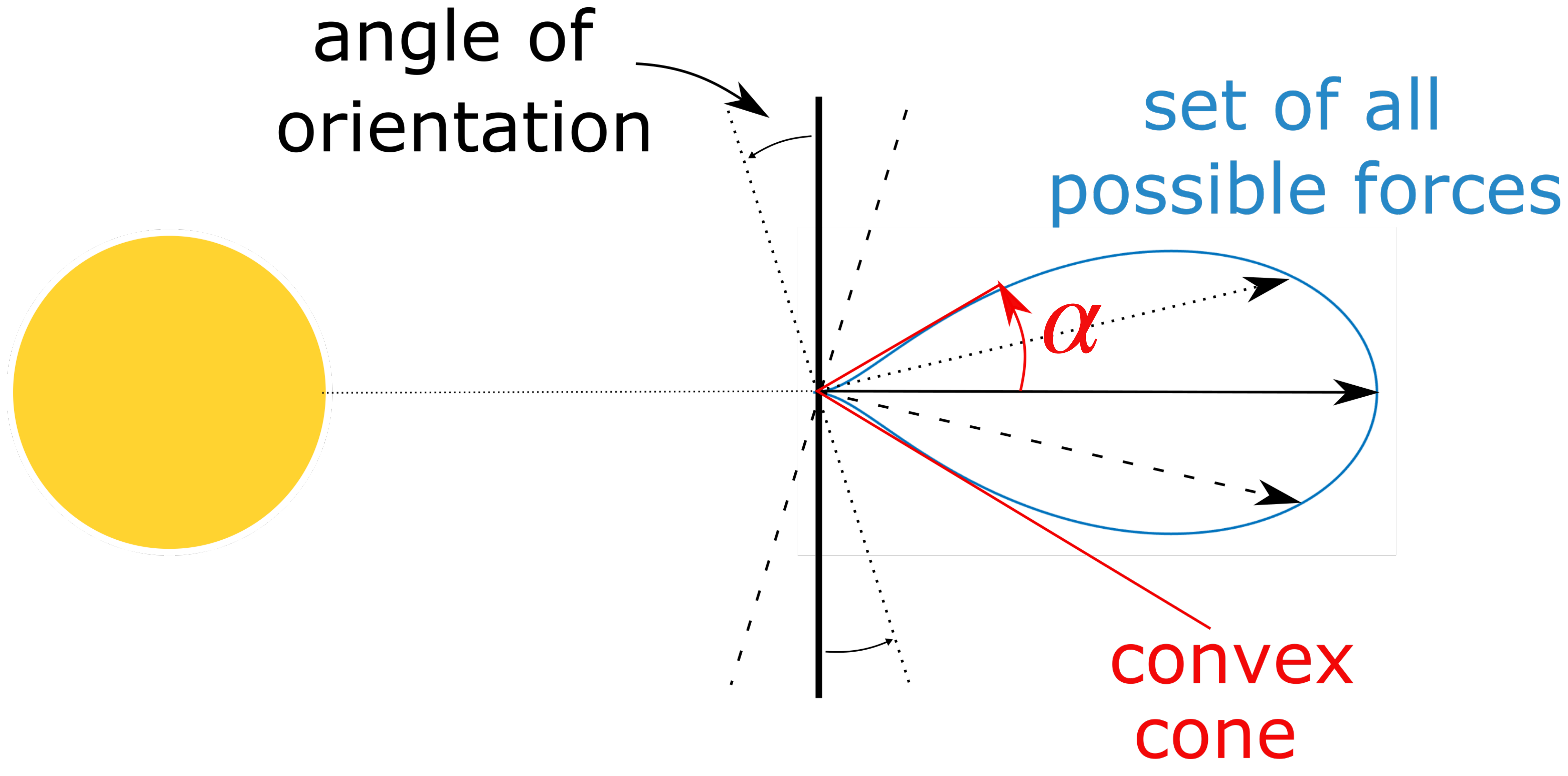
trigonometric polynomial of degree 4

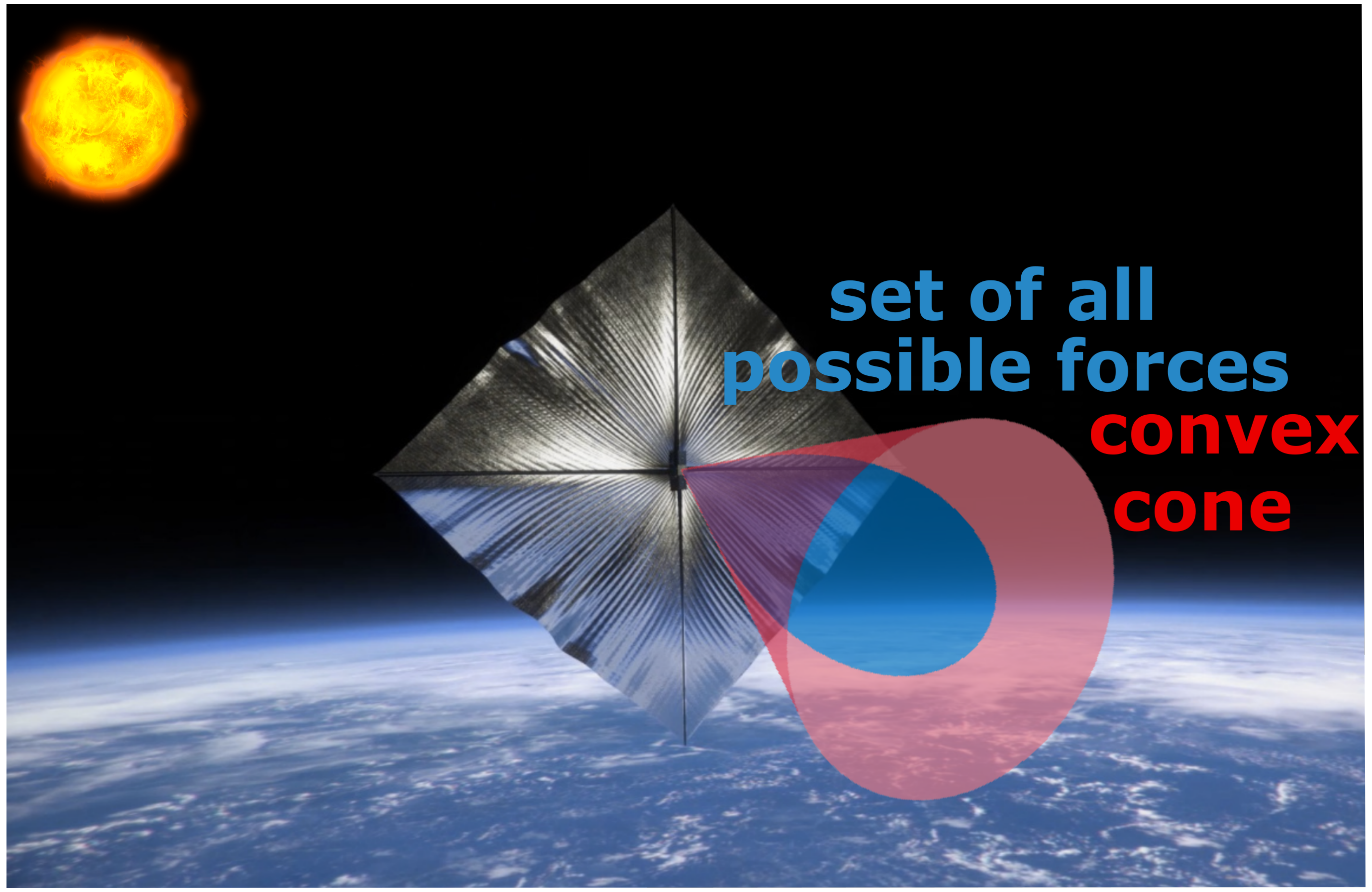



max number of roots = **8** $\iff$ max number of bangs = **5**

# 2. Convexification of the control set

$$\boldsymbol{u} \in \mathcal{U} \subset K_\alpha := \operatorname{cone}(\mathcal{U})$$



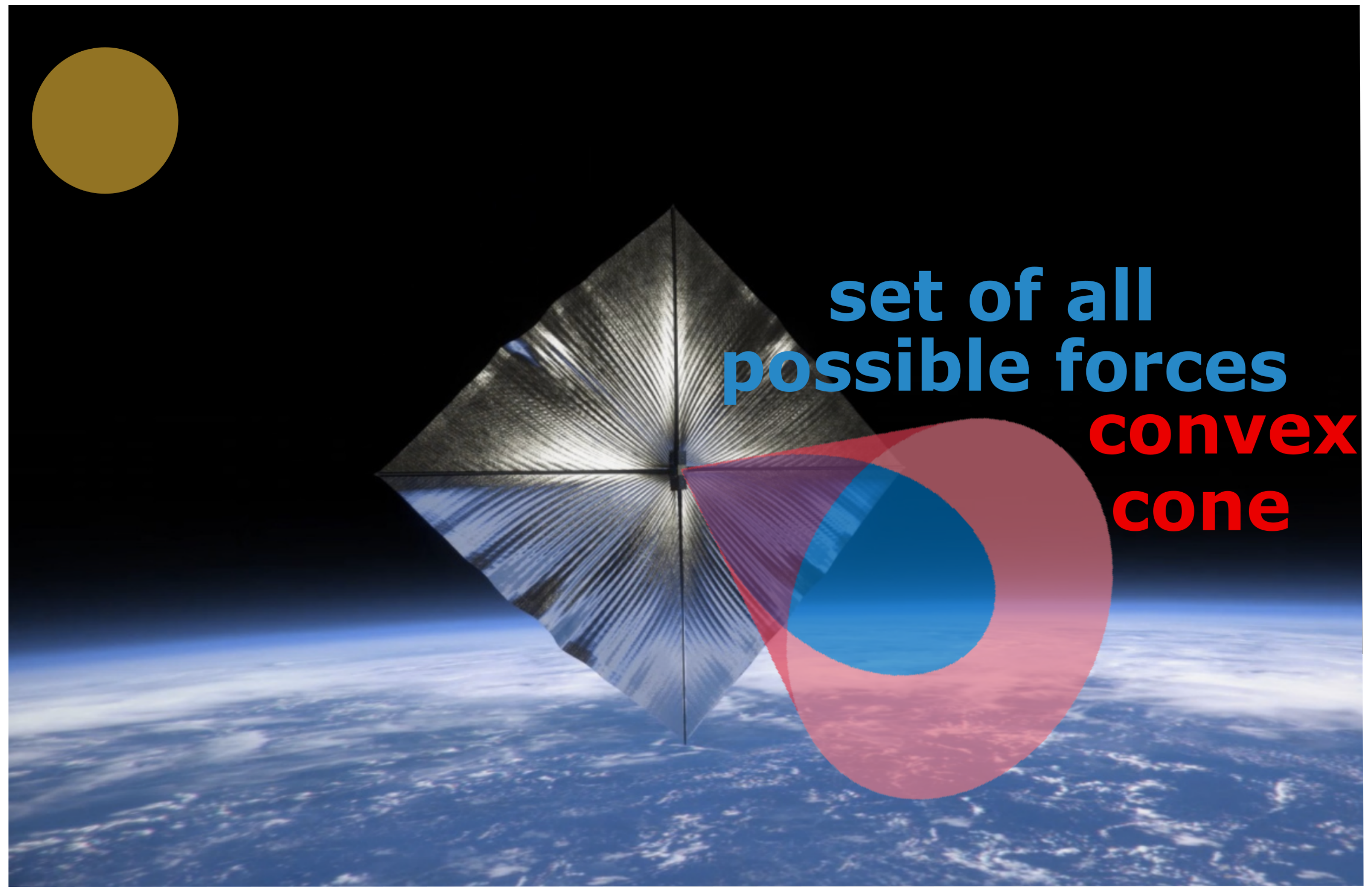
# 2. Reliable initial guess from convex optimization

$$\max_{\cancel{u(f)\in\mathcal{U}}} \ \delta I(2\pi)\cdot d_I \qquad \text{subject to} \qquad \delta I(2\pi) = \varepsilon \int_{\mathbb{S}^1} \sum_{i=1}^{3} u_i F_i(I,f)\,\mathrm{d}f$$

$$\delta I(0) = 0, \qquad \delta I(2\pi) \parallel d_I$$

$$\downarrow$$

$$\max_{u(f)\in\partial K_\alpha}$$



# 2. Numerical solution of the semi-infinite problem

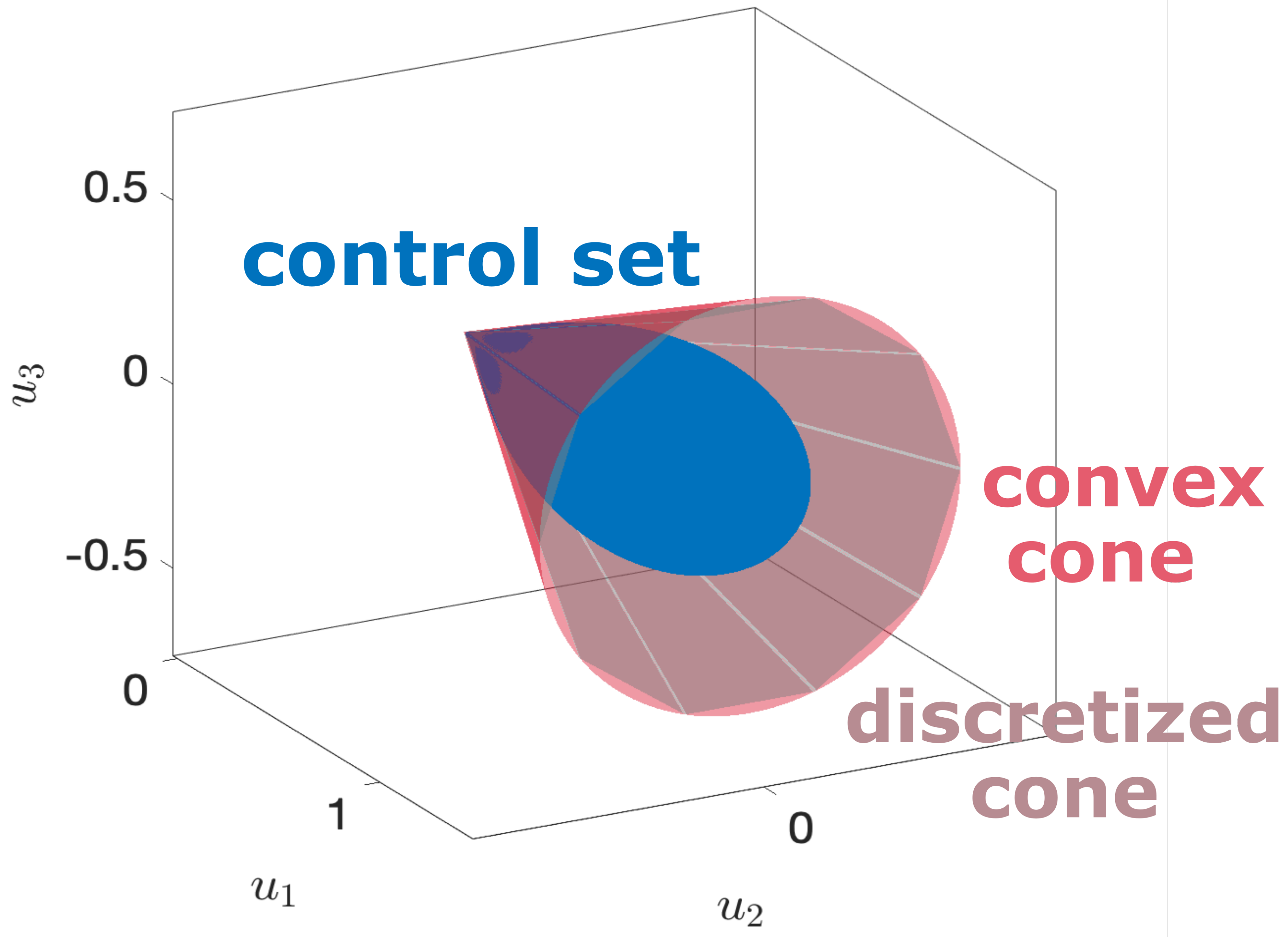
**Parametrization of the cone**

Controls are combinations of

generators $u(f) = \sum_j \gamma_j(f) V_j$

$\gamma_j(f) \geq 0,\ f \in \mathbb{S}^1$

**Fourier series** of the dynamics and $\gamma_j(f)$ in $f$



# 2. Numerical solution of the semi-infinite problem

**Constraint** $\gamma_j(f) \geq 0 \longrightarrow$ **positive polynomials [Nesterov, 2000]**

Leverage on formalism of squared functional systems

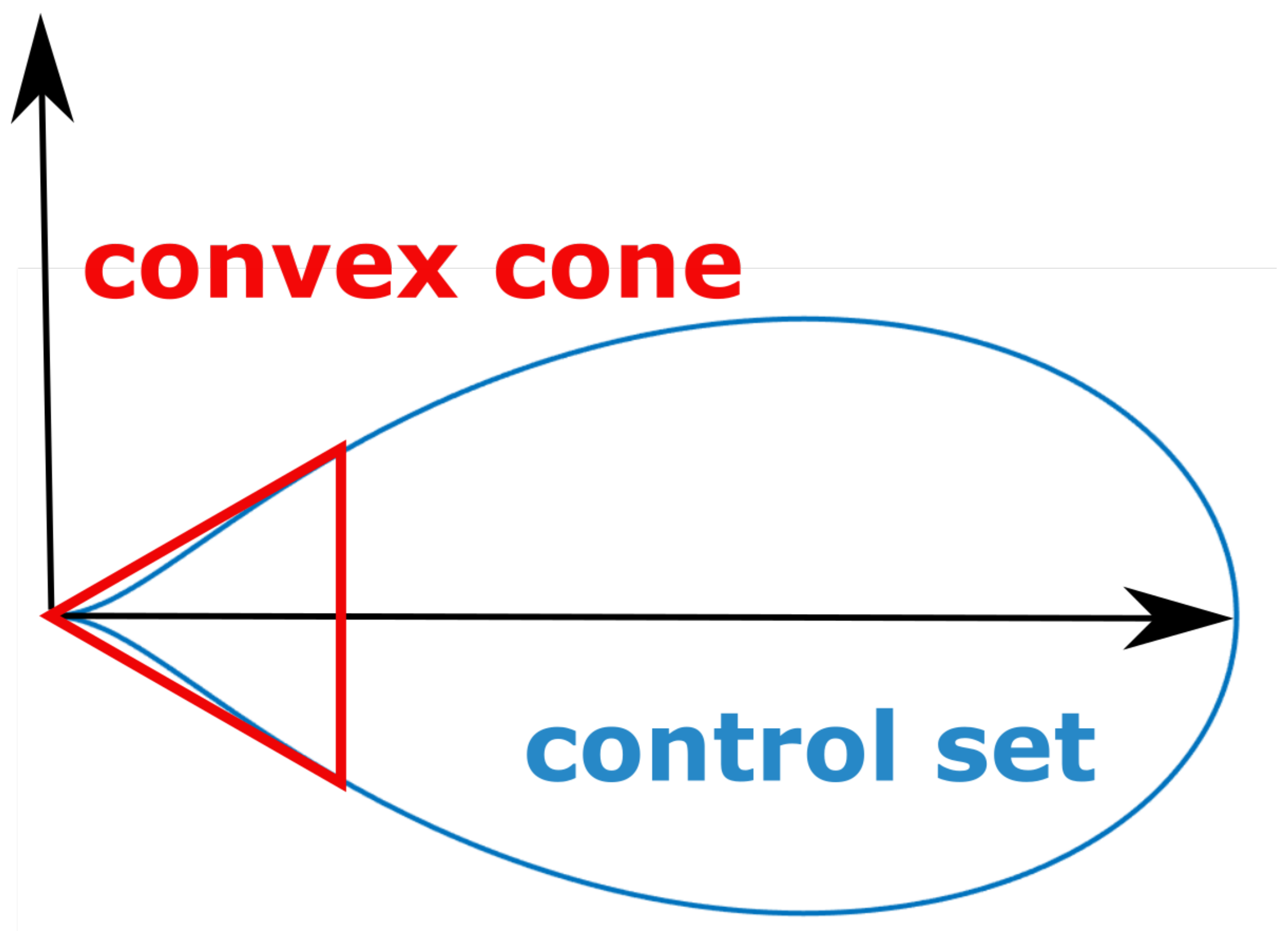
$$\downarrow$$

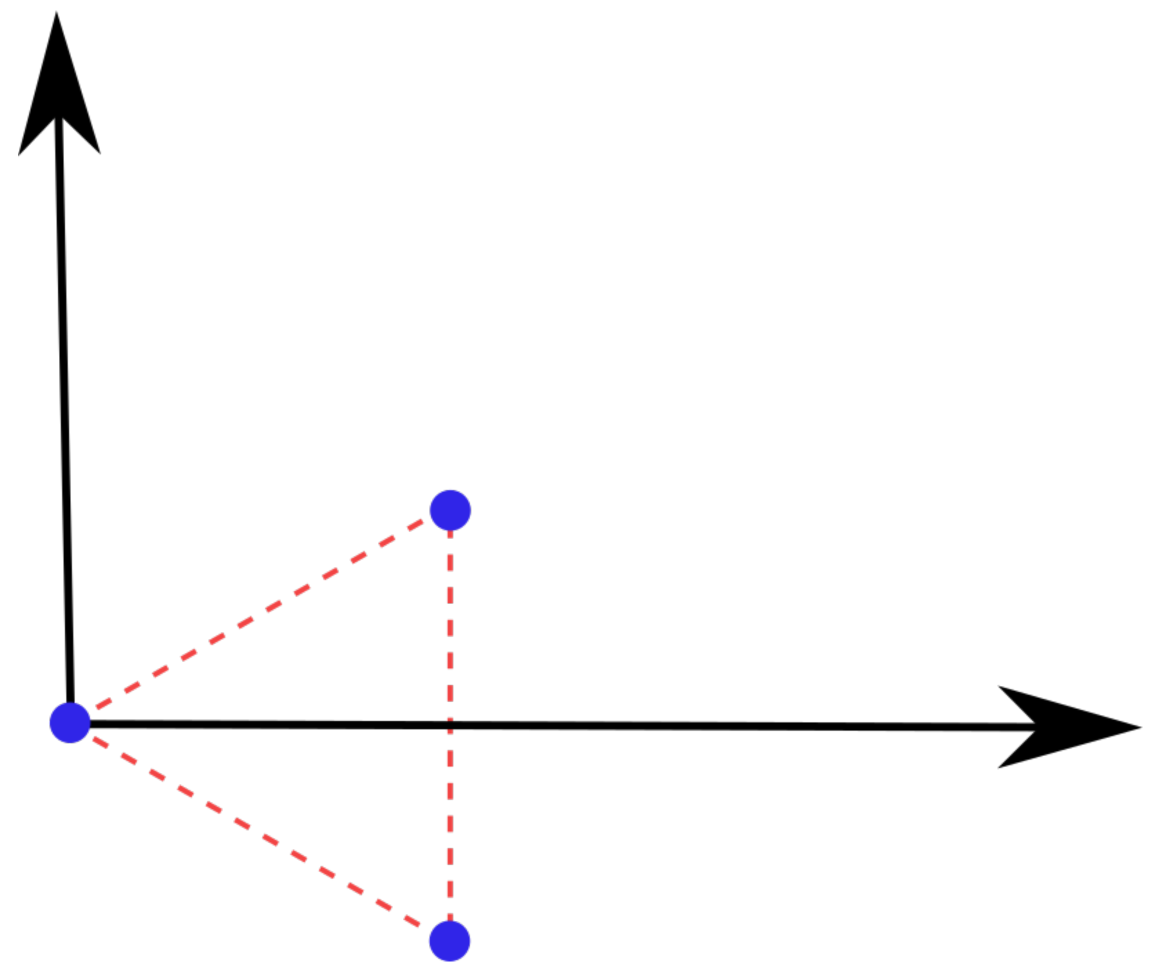
**LMI**

$$\downarrow$$

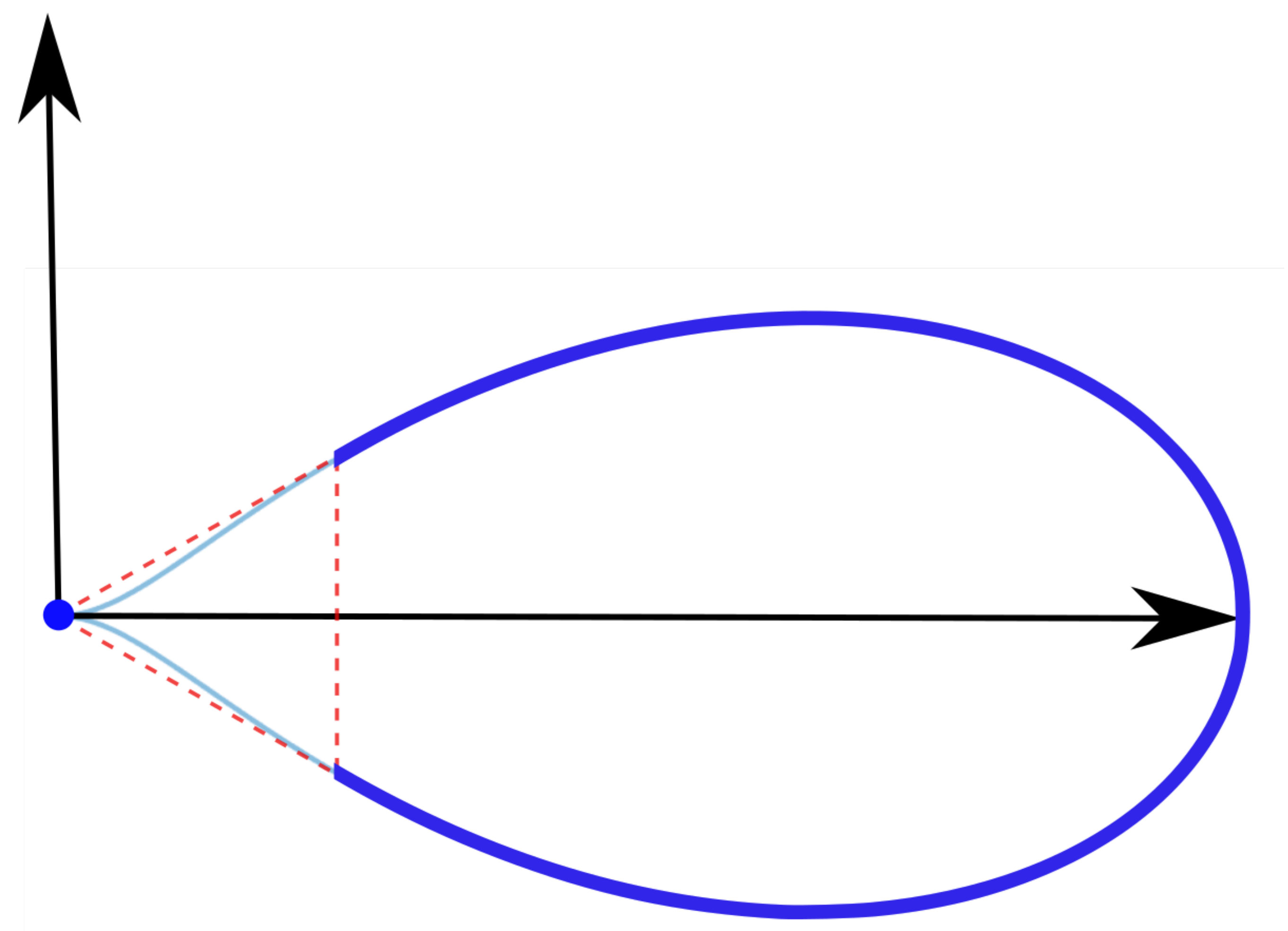
convex programming

# 2. Possible solutions



Convex cone



Real control set

# 3. Algorithm of solution of the OCP

1. Convex optimization: structure of the control, initial guess for the co-state.

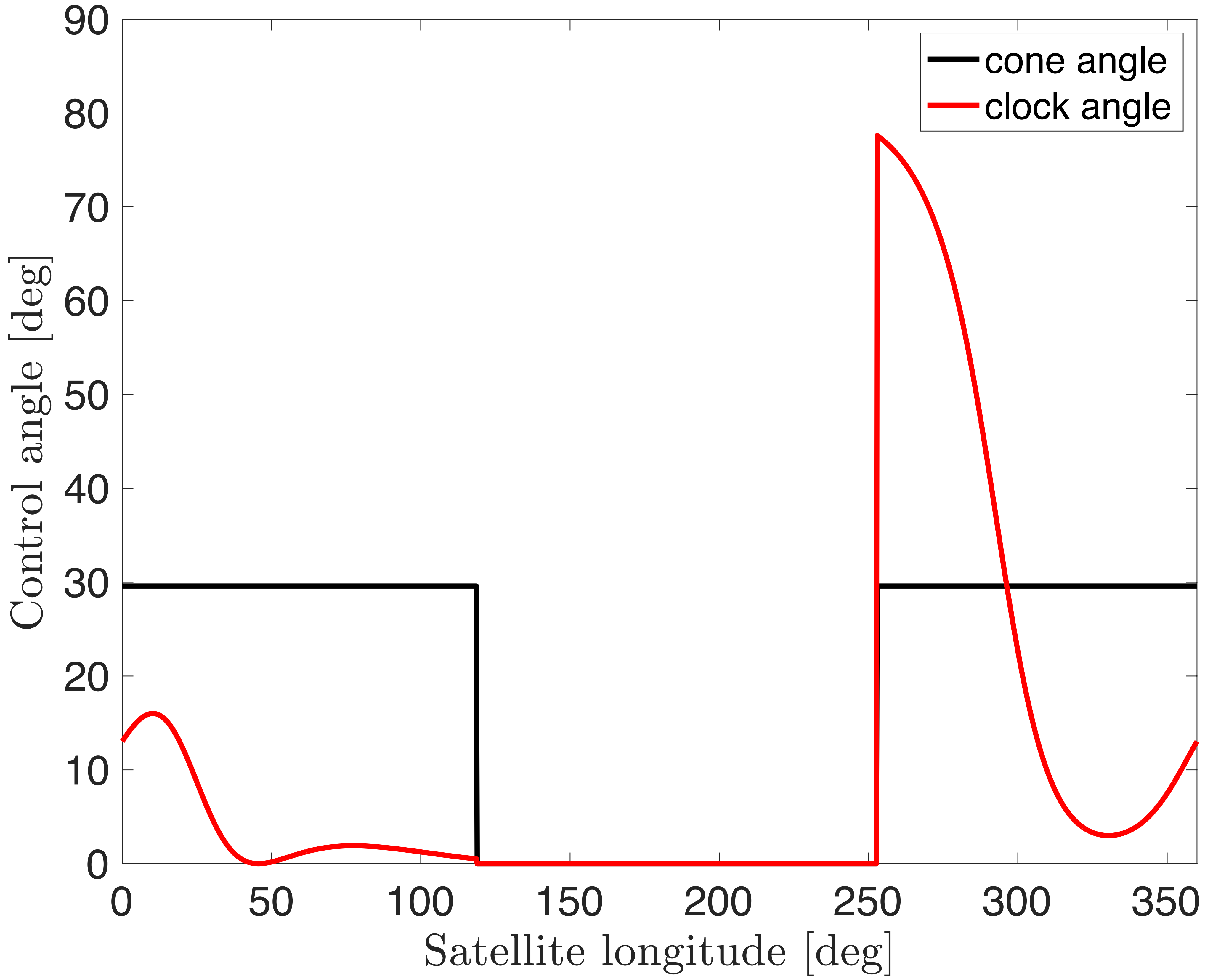
2. Multiple shooting for a given control structure, using *control toolbox*.

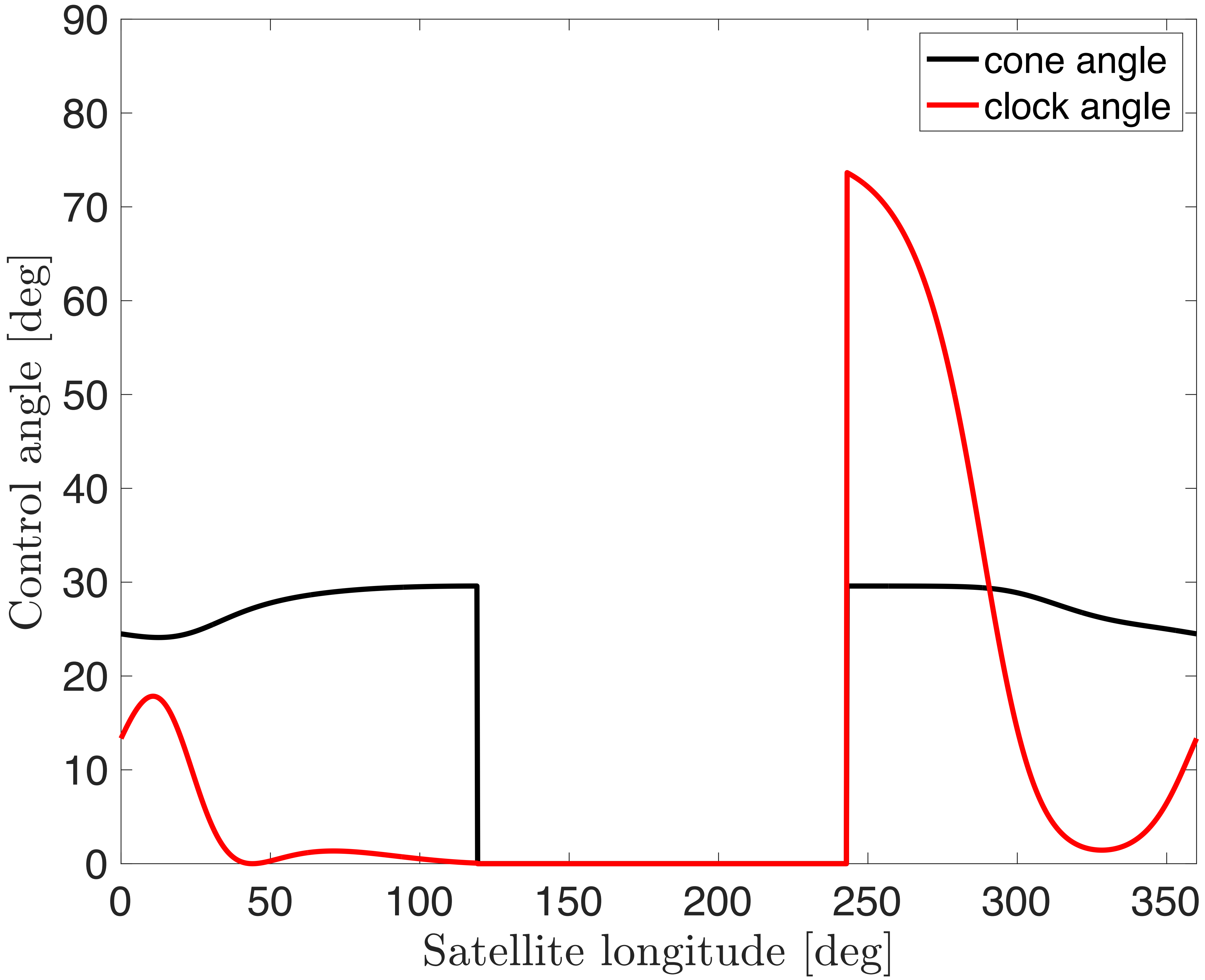
3. Homotopy to the real control set.

https://ct.gitlabpages.inria.fr/gallery/solarsail/solarsail-simple-version.html

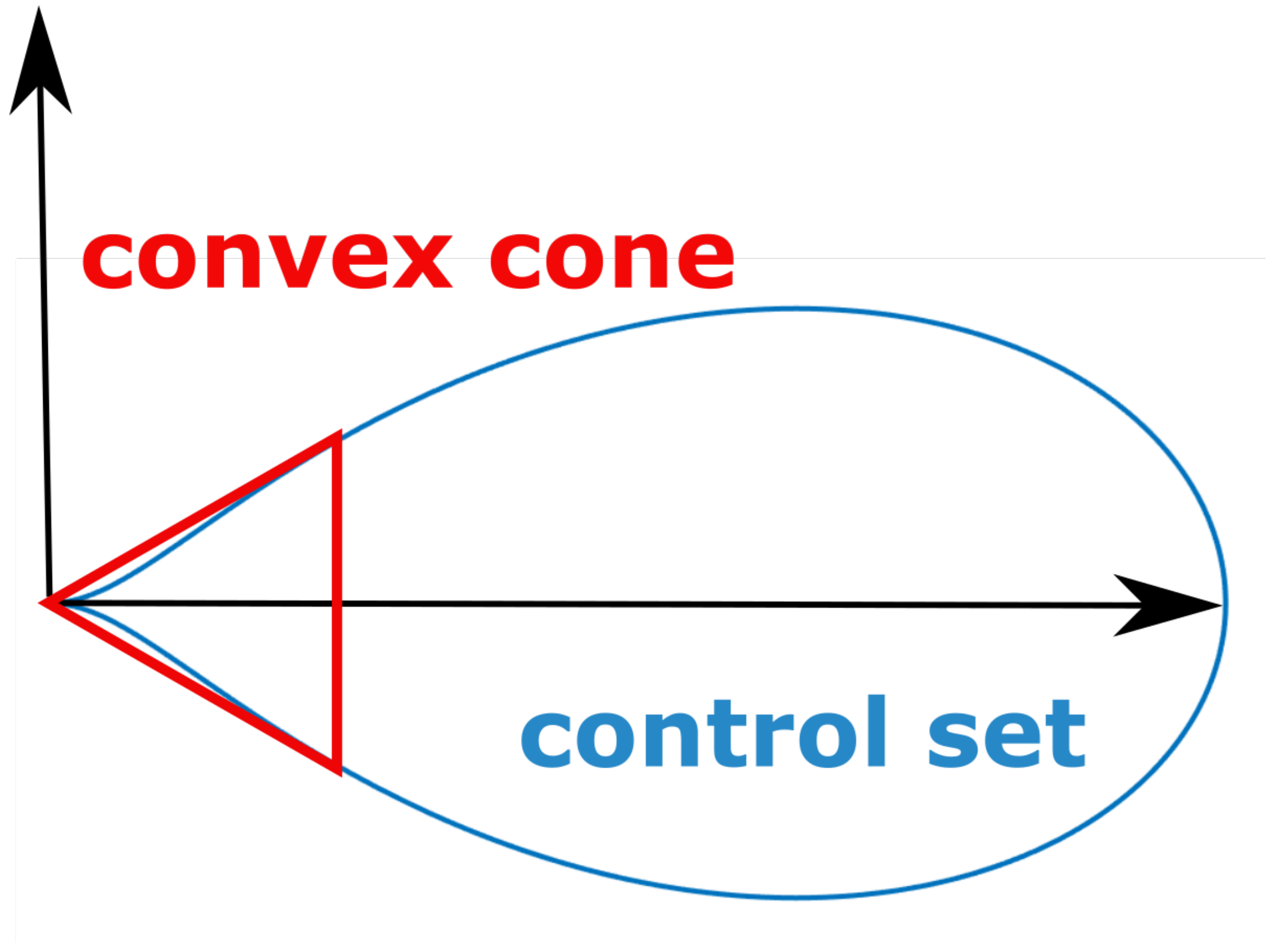
# 4. Case study 1

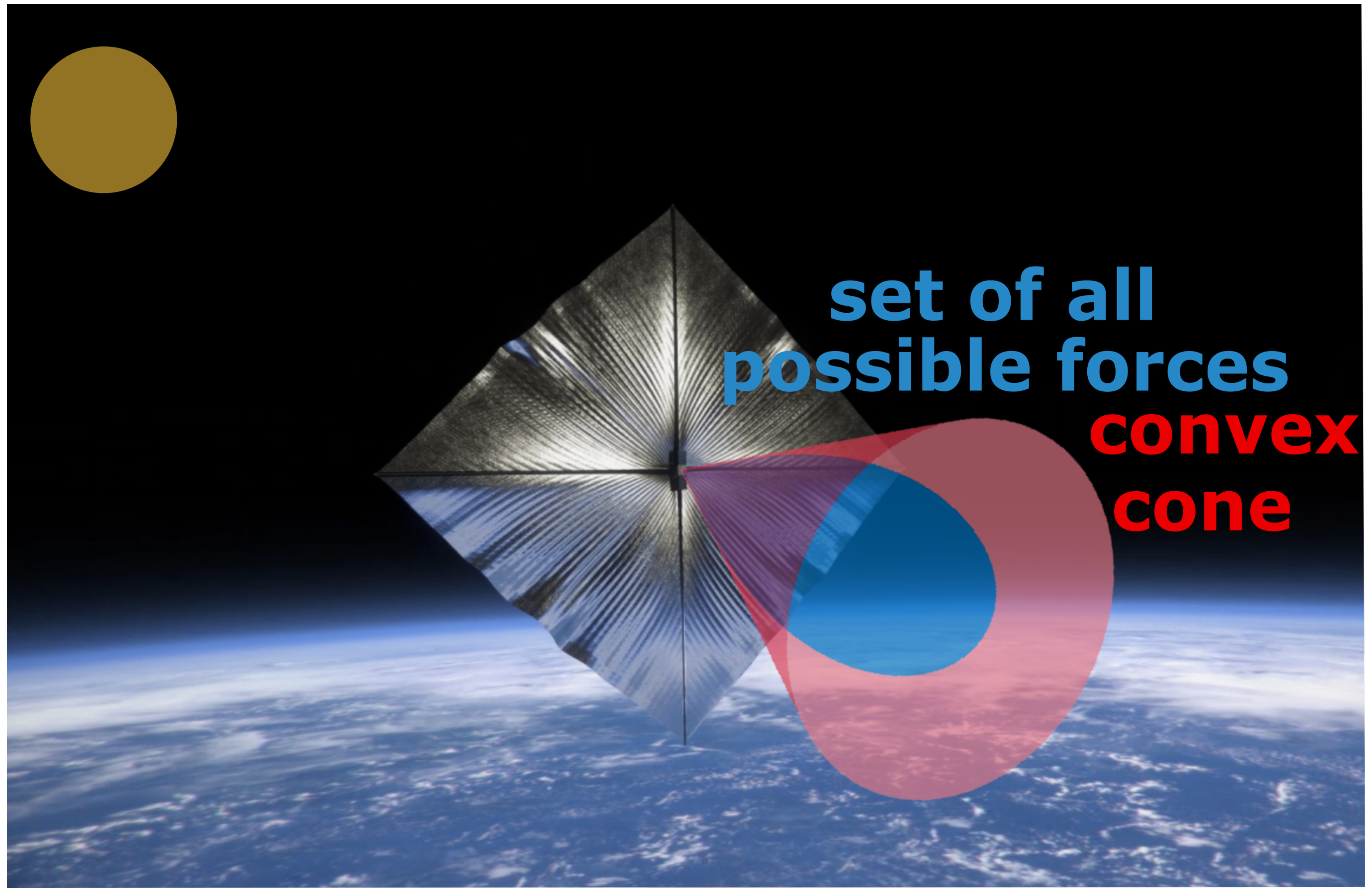
Increase of eccentricity



# 4. Change of structure between initial guess / solution



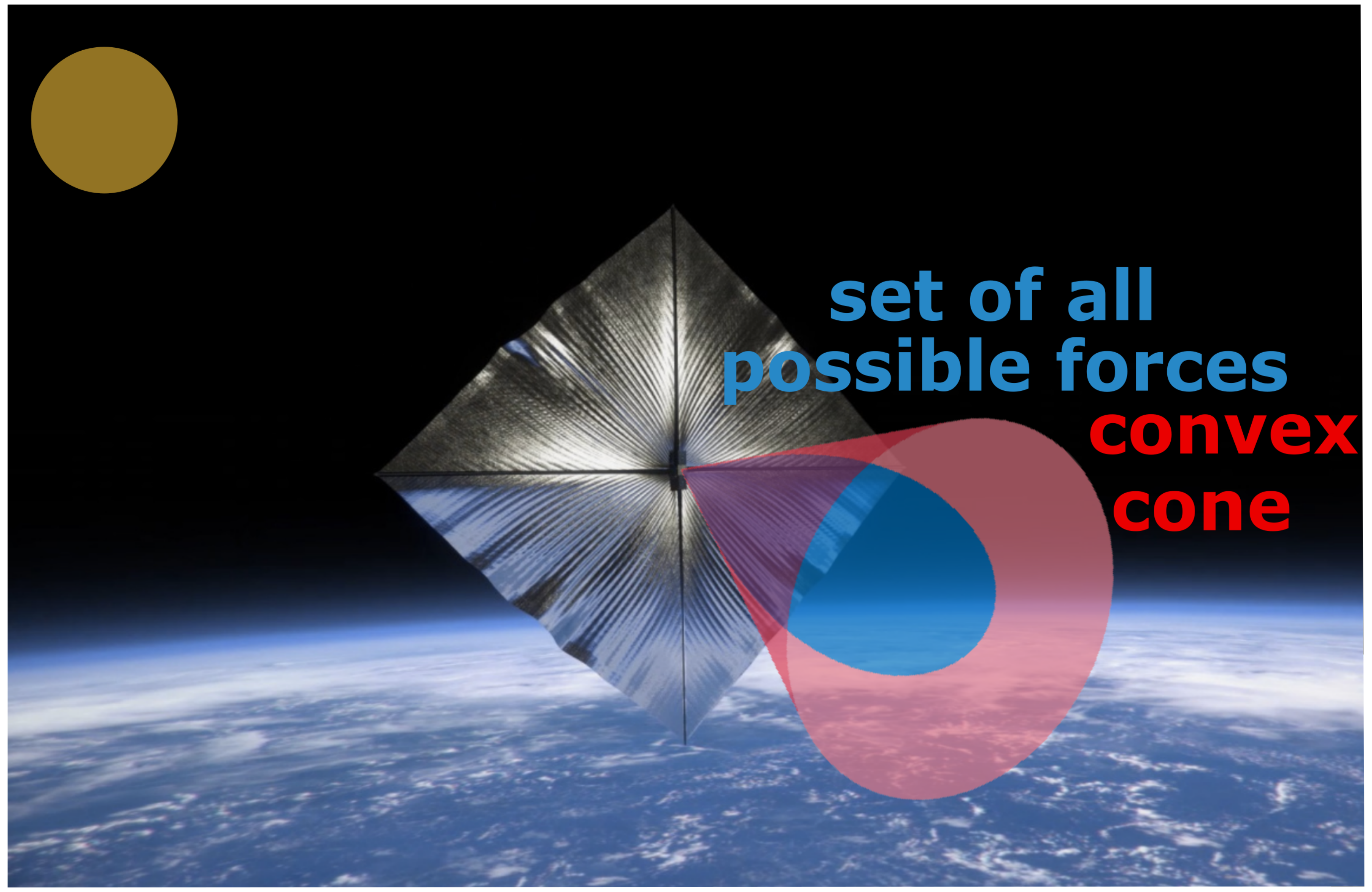
# 4. Updated algorithm

1. Initial guess (co-state + structure)

2. Multiple shooting on bounded cone

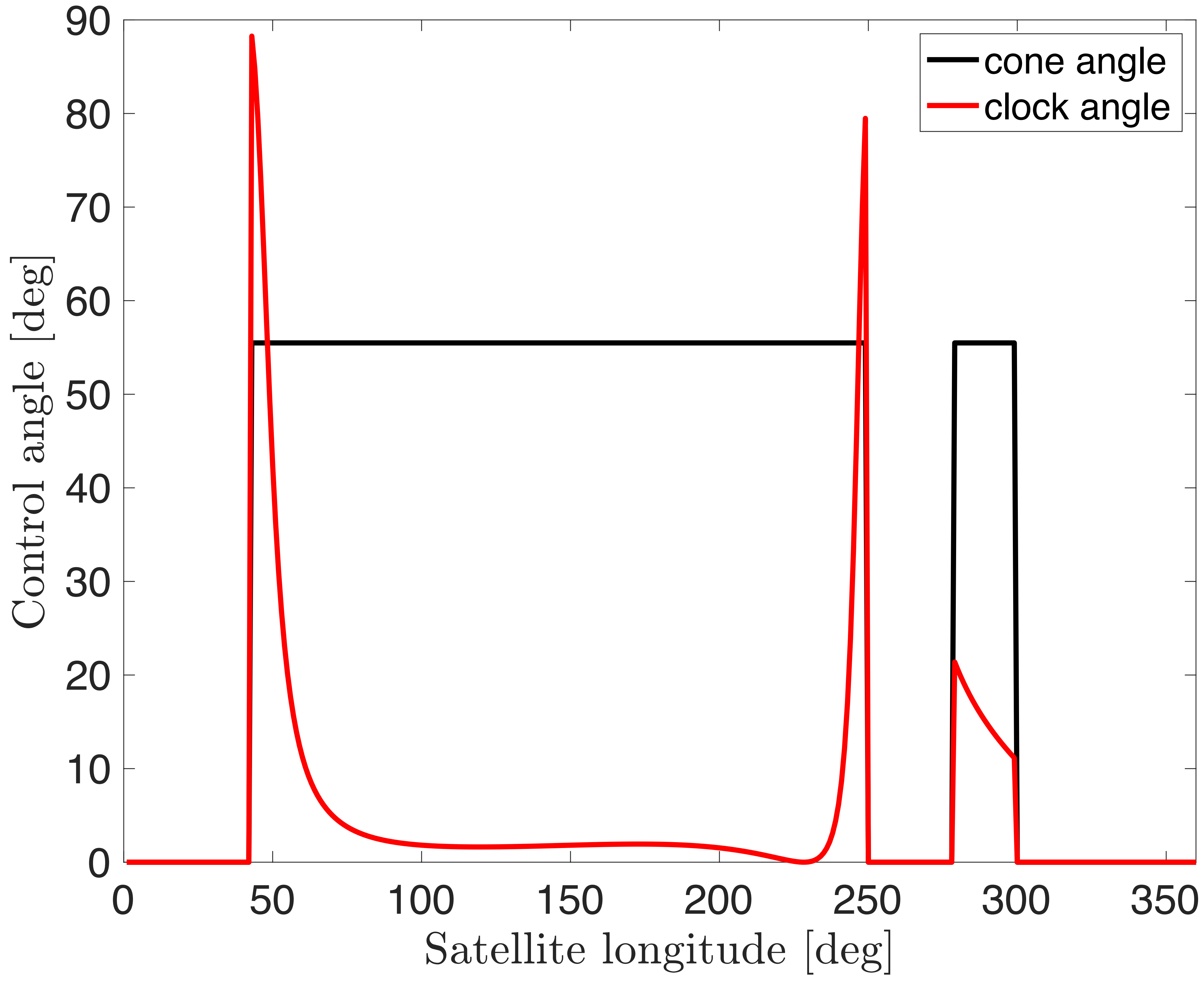
3. Homotopy to the real control set

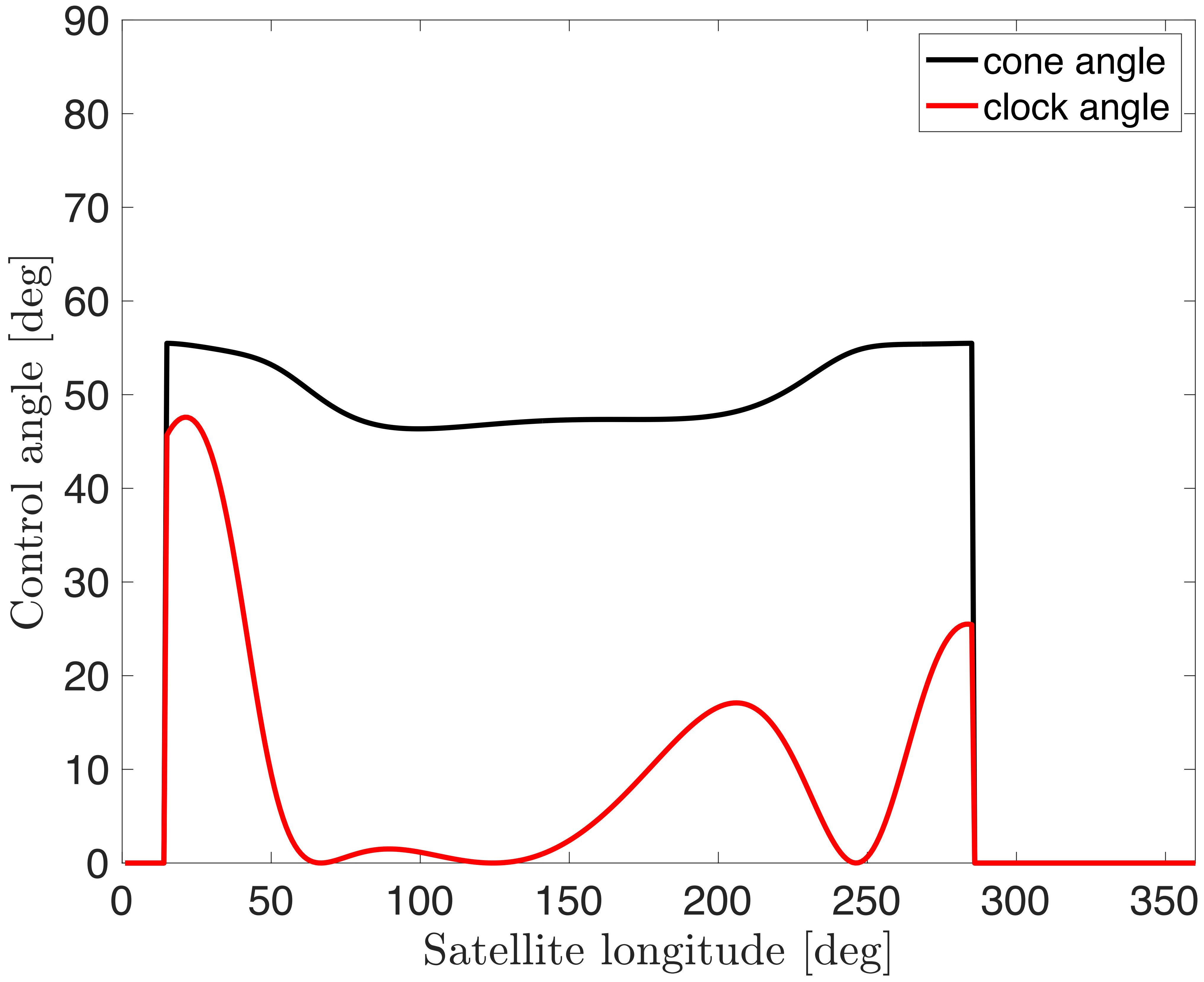
4. Callback to detect change of structure + switch function to initialize new structure



# 4. Case study 2

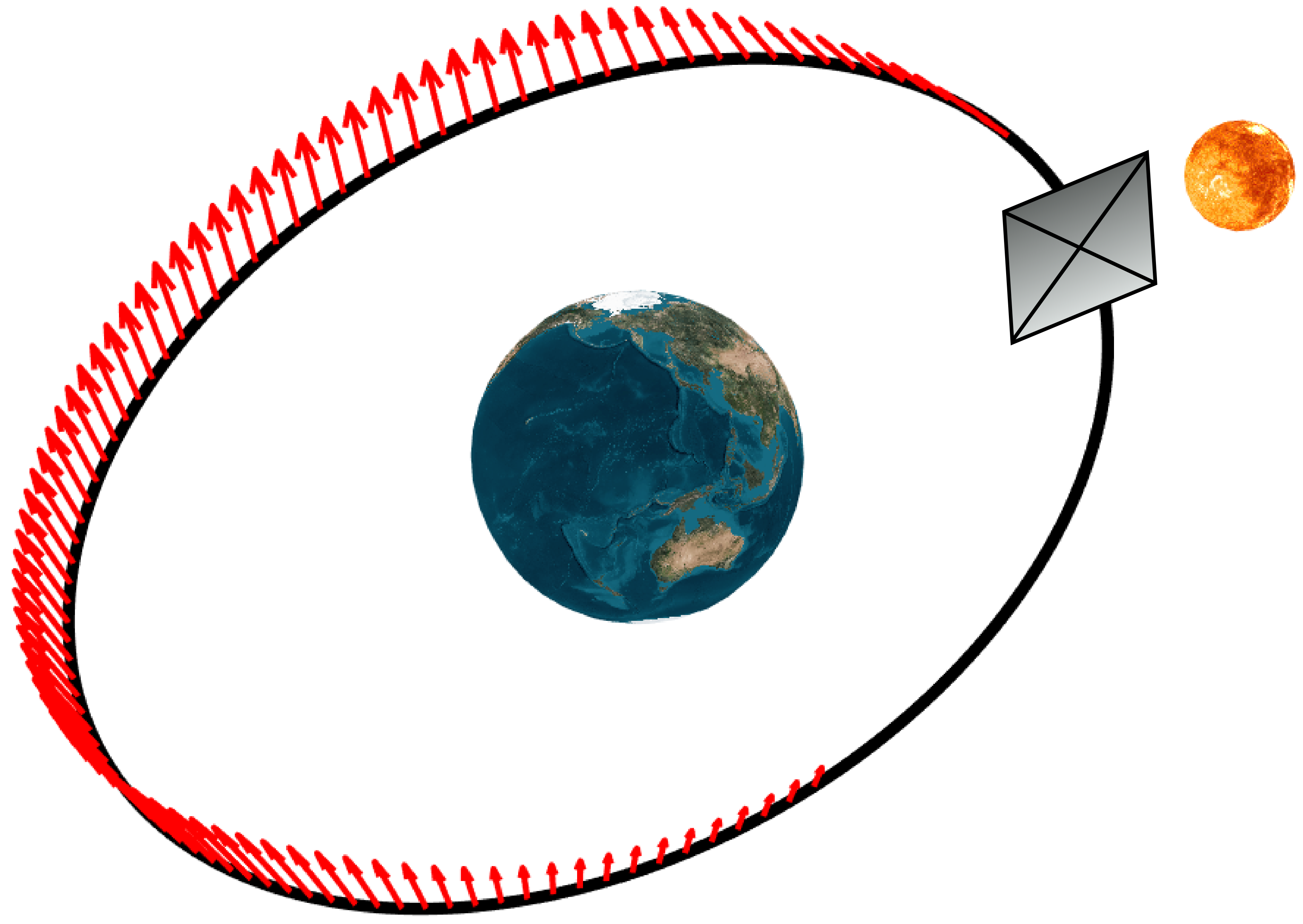
Increase of inclination



# 4. Conclusions

Optimal control algorithm allowing to change the orbit



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# 1. Dynamical system

$$\begin{cases} \dfrac{\mathrm{d}I}{\mathrm{d}t} = \varepsilon \sqrt{\dfrac{a\left(1-e^2\right)}{\mu}} G(I,f)\,u \\ \dfrac{\mathrm{d}f}{\mathrm{d}t} = \omega(I,f) + \varepsilon \sqrt{\dfrac{a\left(1-e^2\right)}{\mu}} G_f(I,f)\,u \end{cases}$$

$$G(I,f) = \begin{pmatrix} 0 & 0 & \dfrac{\sin\left(\omega+f\right)}{\sin i\ (1+e\cos f)} \\ 0 & 0 & \dfrac{\cos\left(\omega+f\right)}{1+e\cos f} \\ -\dfrac{\cos f}{e} & \dfrac{2+e\cos f}{1+e\cos f}\dfrac{\sin f}{e} & \dfrac{\cos\left(\omega+f\right)}{1+e\cos f} \\ \dfrac{2\,a\,e}{1-e^2}\sin f & \dfrac{2\,a\,e}{1-e^2}\left(1+e\cos f\right) & 0 \\ \sin f & \dfrac{e\cos^2 f + 2\cos f + e}{1+e\cos f} & 0 \end{pmatrix}$$

$$\omega(I,f) = \sqrt{\frac{\mu}{a\,(1-e^2)^3}}(1+e\cos f)^2,$$

$$G_f(I,f) = \begin{pmatrix} \dfrac{\cos f}{e} & -\dfrac{2+e\cos f}{1+e\cos f}\dfrac{\sin f}{e} & 0 \end{pmatrix}$$

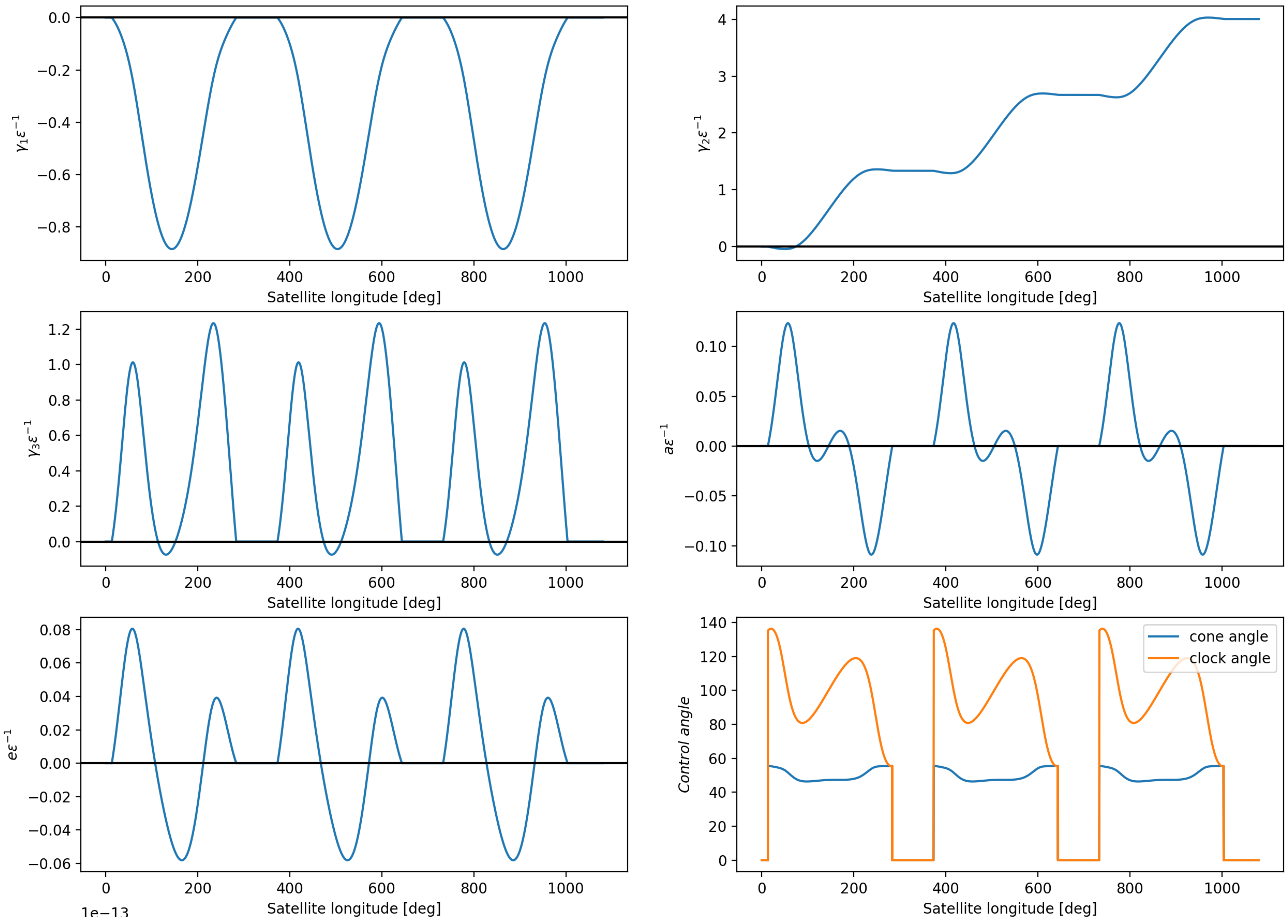
# 1. Dynamical system

$$I := \bar{I} \qquad\qquad \frac{\mathrm{d}f}{\mathrm{d}t} = \omega(\bar{I}, f)$$

$$\frac{\mathrm{d}I}{\mathrm{d}f} = \frac{\varepsilon}{\omega(\bar{I}, f)} \sqrt{\frac{\bar{a}\left(1 - \bar{e}^2\right)}{\bar{\mu}}} G(\bar{I}, f)\, u(f)$$

$$\delta I = I - I_0$$

$$\frac{\mathrm{d}\delta I}{\mathrm{d}f} = \frac{\varepsilon}{\omega(\bar{I}, f)} \sqrt{\frac{\bar{a}\left(1 - \bar{e}^2\right)}{\bar{\mu}}} G(\bar{I}, f)\, u(f)$$



$\gamma_1\varepsilon^{-1}$
$\gamma_2\varepsilon^{-1}$
$\gamma_3\varepsilon^{-1}$
$a\varepsilon^{-1}$
$e\varepsilon^{-1}$
1e−13
Control angle
cone angle
clock angle
Satellite longitude [deg]