On semi-parametric estimation in monotone single-index copulas

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Since Sklar's Theorem [4], a joint distribution with continuous margin can be decomposed into marginal distributions and a copula which explains the dependance structure among random variables. Copulas are well-known in various fields of applied mathematics, such as marketting [3], finance [2], biology [6] and medecine [5].

Applications above involve covariates $X \in \mathbb{R}^d$ to explicit the dependance structure of a vector $Y = (Y_1, \dots, Y_p) \in \mathbb{R}^p$. The relationship between these covariates and the marginal distribution of Y_i denoted by F_i can be modelled by a monotone single-index copula C of the form

$$C((F_1(Y_1), \dots, F_p(Y_p))|\Psi_0(\alpha_0^T X)),$$

where α_0 and Ψ_0 are unknown. Moreover, for identifiability conditions, α_0 belongs to the $d-1$ unit dimensional sphere and Ψ_0 is monotonic as made by [1].

The main aim of my work is to estimate α_0 and Ψ_0 under the monotonicity assumption in the high-dimensional context, i.e. when d is allowed to depend on n and to grow to infinity with n. To address this issue, I study two different M-estimation procedures : a pseudo-maximum likelihood procedure and a weighted minimum distance procedure. Both procedures consist in minimizing an appropriate criterion over $\mathcal{S} \times \mathcal{M}$, where $\mathcal{S}$ is a given closed subset of the $d-1$ unit dimensional sphere, and $\mathcal{M}$ is the set of all non-decreasing real valued functions. In the particular case where $\mathcal{S}$ is the whole unit sphere, I establish the rate of convergence for the regression function $\Psi_0(\alpha_0^T \dot{)}$ estimator of both procedures in the L^2-distance.

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