Neumann approximate and null boundary controllability for semilinear heat equations

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The aim of this work is to study the boundary null and approximate controllability problem for a semilinear heat equation with Neumann boundary conditions. We approach this using new Carleman estimates to establish the observability inequality for the adjoint system. We show that the system is is null and approximately controllable by using controllability results for the linearized problems and we finish the paper by using the fixed point method to solve the semilinearity.

Let $\Omega \subset \mathbb{R}^N$ where $N \in \mathbb{N}\backslash\{0\}$, be a bounded subset with boundary Γ of class C^∞ and let $T > 0$, $Q = \Omega \times (0,T)$, $\Sigma = \Gamma \times (0,T)$ and $\Gamma_0 \subset \Gamma$ an open and nonempty subset. Let $\mathcal{O}$ be an observatory and γ be a control zone both on a part of the boundary Γ excluding Γ_0, which means that $\mathcal{O}$ and γ are two nonempty subsets of $\Gamma \setminus \Gamma_0$ with possibly nonempty intersection. We consider the semilinear heat equation with linear Neumann boundary conditions :

$$\begin{cases} \dfrac{\partial y}{\partial t} - \Delta y + f(y) & = & 0 & \text{in} & Q, \\ \dfrac{\partial y}{\partial \nu} & = & h_0 \chi_{\mathcal{O}} + w \chi_\gamma & \text{on} & \Sigma, \\ y(\cdot, 0) & = & y^0 & \text{in} & \Omega. \end{cases} \tag{1}$$

The null controllability problem can be stated as follows : *Given* $T > 0$, $h_0 \in L^2(\mathcal{O} \times (0,T))$, $y^0 \in L^2(\Omega)$*, find a control* $w \in L^2(\gamma \times (0,T))$ *such that if* $y \in L^2((0,T); H^1(\Omega))$ *is solution of* (1) *then*

$$y(w; ., 0) = 0 \ \textit{in} \ \Omega. \tag{2}$$

Théorème 1. *Let Ω be a open bounded subset of $\mathbb{R}^N$ with boundary Γ of class C^∞, f be a globally Lipschitz function of class C^1. Then, there exists a positive function θ later on such that for any $h_0 \in L^2(\mathcal{O} \times (0,T))$ satisfying $\theta h_0 \in L^2(\mathcal{O} \times (0,T))$ and any $y^0 \in L^2(\Omega)$, we can find a control $w \in L^2(\gamma \times (0,T))$ such that the solution $y = y(w)$ of* (1) *satisfies* (2)*. Moreover, the control w can be chosen such that*

$$\|w\|_{L^2(\gamma \times (0,T))} \leq C \left(\|y^0\|_{L^2(\Omega)} + \|\theta h_0\|_{L^2(\mathcal{O} \times (0,T))} \right), \tag{3}$$

where $C = C(\Omega, \gamma, \alpha, \|a_0\|_{L^\infty(Q)}, T) > 0$, with a_0 the Lipschitz coefficient of the function f.

Our article has not yet been published.

Références

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