On a shock capturing finite volume method that can solve fully incompressible flows

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In Computational Fluid Dynamics, finite volume methods are often specifically designed for shock waves [3], low Mach flows [1], or incompressible flows [2]. However, in an industrial context, the flow can include both high Mach and low Mach regions and transition in time between the two regimes. Furthermore the engineer may want to simulate a fully incompressible flow, for instance when the fluid equation of state is not easily available.

We propose a new finite volume discretisation that can precisely capture shock waves in high Mach flows, and is well defined when the fluid is assumed incompressible, ie where the Mach number is everywhere exactly zero. This new method is strongly inspired by implicit in time finite volume schemes on staggered grids that were historically designed for incompressible flows [2]. These staggered discretisations are popular among multiphase flow engineers for their correct low Mach number asymptotic expansion [6]. However, they are generally non conservative, and their stability analysis has historically been performed with a heuristic approach and the tuning of numerical parameters ([5]).

In this poster, we first investigate the linear L^2-stability of staggered schemes by analysing their numerical diffusion operator. We further study the connection between the numerical diffusion operator on one hand and both the low Mach number precision and checkerboard oscillations on the other hand. We then propose a new conservative linearly L^2-stable finite volumes schemes for both compressible and incompressible fluid models. Unlike [4], our scheme is based on a carefully chosen numerical diffusion operator and the proof of stability follows from the symmetrisation of the system. An important remark is that unlike Godunov type schemes, the numerical diffusion operator of a symmetric system is not symmetric. This property is fundamental to ensure low Mach precision and avoid spurious checkerboard modes oscillations. Indeed, a Total Variation analysis shows that it is the skew-symmetric part of the diffusion matrix that allows for the control of the spurious oscillations.

We give some numerical results showing the good behaviour of the method in 1D and 2D for incompressible flows, low Mach and high Mach number flows, including shock waves.

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