Analysis of staggered finite volume methods on general meshes for incompressible Navier-Stokes problems

Pierre-Loïc BACQ, CEA Saclay, DES/ISAS/DM2S/LMEC - Gif-sur-Yvette
Antoine GERSCHENFELD, CEA Saclay, DES/ISAS/DM2S/LMEC - Gif-sur-Yvette
Michael NDJINGA, CEA Saclay, DES/ISAS/DM2S/LMEC - Gif-sur-Yvette

The development of numerical solutions for fluid dynamics problems is an important area of research. Nuclear safety, for instance, requires to compute complicated dynamics, where the flow is multiphase and compressible. However, developing a reliable numerical scheme for one-phase incompressible flow is crucial. On one hand, it is an essential building block for further models and on the other hand, dificulties with more complete models can be traced back to this more simple model.
Spatial discretisations of this system usually favour staggered discretisations which is long known to ensure robustness and to avoid spurious modes [7]. However, for most thermal hydraulic codes applied in the nuclear field (TRACE, CATHARE, ...[2]), MAC-like schemes require the use of cartesian grids, which can be quite restrictive in practice. To get beyond those restrictions, the thermal hydraulics service of the CEA Saclay developed a family of numerical schemes called PolyMAC which generalizes MAC schemes to general meshes.
All PolyMAC schemes result from the synthesis of several classical approaches for scalar advection-diffusion models, like "Hybrid Mixed Mimetic" [6], "MPFA-O" [1] or "Compatible Discrete Operators" [3] and they all have been implemented in the open source software TRUST [5].
In this presentation, we will detail how the different versions of PolyMAC are defined, how they differ from each other and what are their respective strengths and weaknesses. To do so, we define a benchmark of 2D and 3D (Navier-)Stokes problems based on the successive benchmarks defined during the cycle of FVCA conferences [4]. Based on the results obtained on the benchmark, we propose guidelines for the optimal use of the different versions of PolyMAC.
Finally, we also identify causes for the weaknesses of each current version of PolyMAC and we propose some modifications to improve the performances of the current implementations.

[1] I. Aavatsmark. *An Introduction to Multipoint Flux Approximations for Quadrilateral Grids.* Computational Geosciences, **6**, 405–432, 2002.

[2] D. Bestion. *11 - the structure of system thermal-hydraulic (sys-th) code for nuclear energy applications.* In F. D'Auria, ed., *Thermal-Hydraulics of Water Cooled Nuclear Reactors*, pp. 639–727. Woodhead Publishing, 2017. doi :https://doi.org/10.1016/B978-0-08-100662-7.00011-7.

[3] J. Bonelle. *Compatible Discrete Operator Schemes on Polyhedral Meshes for Elliptic and Stokes Equations.* Ph.D. thesis, Université Paris-Est, 2014.

[4] F. Boyer, P. Omnes. *Benchmark proposal for the fvca8 conference : Finite volume methods for the stokes and navier–stokes equations.* In C. Cancès, P. Omnes, eds., *Finite Volumes for Complex Applications VIII - Methods and Theoretical Aspects*, pp. 59–71. Springer International Publishing, Cham, 2017.

[5] CEA. *TRUST platform.* `https://github.com/cea-trust-platform`, 2022.

[6] J. Droniou, R. Eymard, T. Gallouët, C. Guichard, R. Herbin. *The gradient discretisation method* . Springer International Pulishing AG, 2018.

[7] F. H. Harlow, J. E. Welch. *Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface.* The Physics of Fluids, **8(12)**, 2182–2189, 1965. doi :10.1063/1.1761178.

Contact : `pierre-loic.bacq@cea.fr`