# Practical resolution of nonlinear PDEs on noisy quantum processors

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We consider a Bose-Einstein condensate constituted with bosons in a harmonic potential $V : \mathbb{R} \to \mathbb{R}$, $x \mapsto V_0 x^2$, with $V_0 > 0$, and denote by $\phi$, the wave function describing the ground state of the condensate. The stationary Gross-Pitaevskii equation consists in a nonlinear eigenvalue problem, which reads

$$
\left|
\begin{aligned}
&\text{Find } (\mu, \phi) \in \mathbb{R} \times H^1(\mathbb{R}) \text{ such that} \\
&A_\phi \phi = \mu \phi, \\
&\|\phi\|_{L^2(\mathbb{R})} = 1,
\end{aligned}
\right.
\tag{1}
$$

where the operator $A_\phi$ in (1) is defined, for any wave function $\psi$, by

$$
A_\phi \psi = \left( -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2}{\mathrm{d}x^2} + V + \kappa |\phi|^2 \right) \psi,
$$

where $\hbar > 0$ is the reduced Planck constant, $m > 0$ the boson mass and $\kappa > 0$ the interaction strength. This nonlinear eigenvalue problem corresponds to the solution of the minimisation of the associated energy functional $E$ under a norm-1 constraint where $E$ has the form $E(\phi) = \mathcal{K}(\phi) + \mathcal{P}(\phi) + \mathcal{I}(\phi)$, the kinetic, potential and interaction contributions that are respectively given by

$$
\mathcal{K}(\phi) = \frac{\hbar^2}{2m} \int_{\mathbb{R}} |\phi'(x)|^2 \, \mathrm{d}x, \qquad \mathcal{P}(\phi) = \int_{\mathbb{R}} V(x)|\phi(x)|^2 \, \mathrm{d}x, \qquad \mathcal{I}(\phi) = \frac{\kappa}{2} \int_{\mathbb{R}} |\phi(x)|^4 \, \mathrm{d}x.
$$

We propose to solve (1) by minimizing the energy $E$ [1] using quantum computers. To do so, for some $N_q \in \mathbb{N}^*$, we are led to introduce a $N_q$-qubit variational state $|\phi\rangle$ to discretize $\phi$. In [2], the authors compute the energy associated to $|\phi\rangle$ using deep circuits called *quantum nonlinear processing units* (QNPUs). An important drawback of this approach is that on current quantum platform important errors due to noise pollute the computations and they become more important as the length of the circuits becomes larger. In this work, we propose an alternative method making use of the Pauli decomposition of nonlinear operators to compute the discretized energy functional by direct measurements of Pauli operators. We show that, when considering a global depolarisation noise model, our method is less affected by the quantum noise as there is no overhead in the circuit size required to compute the observable. For terms where the Pauli operators in the decomposition commute, we show that we are able to outperform the QNPU method consistently. This is the case for the interaction and potential term, at least in the harmonic case. For the kinetic term, we determine that there exists a number of samples beyond which our method outperforms the QNPU method.

[1] C. M. Dion, E. Cancès. *Ground state of the time-independent Gross-Pitaevskii equation.* Comput. Phys. Commun., **177(10)**, 787–798, 2007. doi:10.1016/j.cpc.2007.04.007.

[2] M. Lubasch, J. Joo, P. Moinier, M. Kiffner, D. Jaksch. *Variational quantum algorithms for nonlinear problems.* Phys. Rev. A, **101(1)**, 010301, 2020. doi:10.1103/PhysRevA.101.010301.

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