High order well-balanced discontinuous Galerkin schemes for the Einstein-Euler equations of general relativity

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During this talk, we will present a novel approach allowing robust and effective numerical simulations of the Einstein-Euler equations: these equations describe the coupled evolution of matter and space-time metric according to the theory of general relativity.
To achieve this result, we propose to rewrite the Z4 formulation of the Einstein field equations, which describes the *space-time* evolution, in a first order strongly hyperbolic form and to couple them with the relativistic Euler equations, which instead model the *matter* evolution on a generic space-time. Then, we discretize the obtained system with a high order ADER discontinuous Galerkin scheme with *a posteriori* sub-cell Finite Volume limiter; moreover, we equip our scheme with so-called *well-balanced* techniques [1, 2]. High order of accuracy allows a precise description of the metric evolution, which is usually smooth, and the further coupling with well-balancing increases the resolution power on small perturbations around equilibria, which otherwise would be hidden by spurious numerical errors. In addition, the use of a Finite Volume limiter guarantees shock-capturing properties, particularly useful in case of mass collision.
The talk will be closed by showing a wide set of numerical results which include standard stability benchmarks for astrophysics as well as the study of an oscillating neutron star and the simulation of a collision between two-puncture black holes.

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[1] M. Dumbser, O. Zanotti, E. Gaburro, I. Peshkov. *A well-balanced Discontinuous Galerkin method for the first order Z4 formulation of the Einstein-Euler system.* In preparation, 2023.

[2] E. Gaburro, M. J. Castro, M. Dumbser. *A well balanced finite volume scheme for general relativity.* SIAM Journal on Scientific Computing, **43(6)**, B1226–B1251, 2021.

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