Mayer optimal control problem on compact Riemannian manifolds under probability knowledge of the initial condition

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We consider an optimal control problem on a compact Riemannian manifold M with imperfect information on the initial state of the system. The lack of information is modeled by a Borel probability measure along which the initial state is distributed. This problem is of fundamental importance both in terms of real world applications and mathematical theory.

Indeed, this layout appears in the modelling of many optimal control problems related to mechanics, robotics or bioreactors. The initial condition is not perfectly known either due to the lack of measurements, errors of measurements, or even due to the nature of the system itself, meaning that the uncertainties are inevitable. As for the theoretical interest, since the optimal control problem with partial information is defined in the 2-Wasserstein space over the Riemannian manifold $\mathcal{P}_2(M)$, we need to define proper tools in this space to describe the problem.

Similar to optimal control problems with perfect information, we introduce the value function of this problem and an associated Hamilton Jacobi Bellman (HJB) equation defined on $\mathcal{P}_2(M)$. We propose to extend the same techniques commonly used in viscosity theory to the space $\mathcal{P}_2(M)$ in order to prove that the value function is the unique viscosity solution to the HJB equation. In particular, we want to define a notion of "smooth" solutions to the HJB equation, define the set of test functions for viscosity super and subolutions, prove a local comparison principle that guarantees uniqueness of the viscosity solution and finally prove that the value function verifies a dynamic programming principle that will guarantee existence of the solution.

The main result of this work is that the value function of the problem is the unique viscosity solution to an HJB equation. The notion of viscosity is defined by exploiting the Riemannian-like structure on $\mathcal{P}_2(M)$.

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