A semi-Lagrangian scheme for first-order HJB equations using neural networks

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We consider a deterministic control problem with finite horizon. On the one hand, the discretization of the Dynamical Programming Principle (DPP) by mesh-based methods suffers from the curse of dimensionality. On the other hand, the discretized DPP is naturally formulated as a chain of optimization problems. Following the ideas of [1] in the stochastic setting, we consider a semi-Lagrangian scheme where the value function at each time step is represented by a neural network. This scheme is applied to an obstacle problem arising from a state-constrained control problem, and some elements of analysis are given in this context. Numerical illustrations are given for dimensions between 2 and 8.

[1] A. Bachouch, C. Huré, N. Langrené, H. Pham. *Deep neural networks algorithms for stochastic control problems on finite horizon : Numerical applications.* Methodology and Computing in Applied Probability, **24(1)**, 143–178, 2022. doi :10.1007/s11009-019-09767-9.

[2] O. Bokanowski, X. Warin, A. Prost. *Neural networks for deterministic HJB equations and application to front propagation with obstacle terms*, 2022.

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