Uncertainty principles in control theory of PDEs

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In this talk we review some classical and recent results relating the uncertainty principles for the Laplacian with the controllability and stabilisation of some linear PDEs. The uncertainty principles for the Fourier transforms state that a square integrable function cannot be both localised in frequency and space without being zero, and this can be further quantified resulting in unique continuation inequalities in the phase spaces. Applying these ideas to the spectrum of the Laplacian on a compact Riemannian manifold, Lebeau and Robbaino obtained their celebrated result on the exact controllability of the heat equation in arbitrarily small time. The relevant quantitative uncertainty principles known as spectral inequalities in the literature can be adapted to a number of different operators, including the Laplace-Beltami operator associated to C^1 metrics or some Schödinger operators with long-range potentials, as we have shown in recent results in collaboration with Gilles Lebeau (Nice) and Nicolas Burq (Orsay), with a significant relaxation on the localisation in space. As a consequence, we obtain a number of corollaries on the decay rate of damped waves with rough dampings, the simultaneous controllability of heat equations with different boundary conditions and the controllability of the heat equation with rough controls.

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